THE MULTI-CUSTOMER LOCAL DELIVERY PROBLEM AND THE SITING OF REPAIR PARTS INVENTORIES

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by

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The Multi-Customer Local Delivery Problem and the Siting of Repair Parts Inventories

by

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I. INTRODUCTION

If all required materials were available at the right time and place for a reasonable price, no manager, business, or government agency would choose to stock them. Unfortunately this is not the case and both the Department of Defense and the Navy maintain large stocks of material in support of their missions. With increasingly complex and specialized weapon systems, the sources of supply are becoming more scarce and procurement lead times are increasing, resulting in the need for increased range and depth of support. Meanwhile pressures to decrease the federal budget deficit and a high inflation rate have often forced the Navy, as well as other government agencies, to operate on budget allotments which may be declining in purchasing power. To maintain previous levels of service, increases in operational efficiency and worker productivity at least equal to that being obtained by private industry are required.

The consolidation of support facilities within the Navy has been one method of improving efficiency. The development of centralized Inventory Control Points (ICPs) have

certainly had significant impact on the supply system. Through the collection and manipulation of a system-wide data base, more intelligent provisioning, outfitting, budgeting, and stockage decisions have been possible. Providing world-wide asset visibility and centralized procurements have also offered improved support at a reduced cost. It is expected further improvements will still be made in this area in the future [Ref. 1].

Much of the success of the ICP effort, however, has to be attributed to the development of high speed communications systems used to transfer information to the ICP and the development of high speed and high capacity computers and peripherals to process this information. Without the necessary capital investment in the above productivity enhancing systems, the ICP would likely be a much less effective and desirable entity.

Consolidations have been occurring in other areas as well. Major stock points at Newport, Rhode Island and Long Beach, California have essentially been closed or consolidated with other support activities. Material for fleet issue has been consolidated at regional Naval Supply Centers (NSCs) located at major demand sources. The most recent moves have been to consolidate wholesale supply support for

several Naval Air Rework Facilities (NARFs) at nearby Naval Supply Centers. Previous support had been provided by Naval Air Station supply departments where those NARFs are located. Since the supply centers often carry material under Defense Logistic Agency (DLA) funding as well as that provided by the Navy Stock Fund and Navy Industrial Fund, stock range and depth should improve over that which was previously available at the air station. This improved stock position should lead to improved point of entry (POE) effectiveness and thus improved customer support, other things being equal. These consolidations of support are made economically more attractive when the supply centers install capital intensive, productivity enhancing automated material handling systems such as NISTARS (Naval Integrated Storage and Retrieval System).

However, by centralizing material at regional centers, distances that material must move after issue to reach the customer may increase substantially. Not only would this possibly increase transportation costs, but more importantly it would likely delay the receipt of required parts on the customer's production line. With components under repair awaiting parts, either test bench or shop space is occupied or maintenance time must be used instead to consolidate the

pieces of the component in progress and store them together until the required parts are received. In either case valuable production resources are lost, thus incurring some delay cost.

II. PREVIOUS STUDIES

The first Naval Air Rework Facility (NARF) wholesale support consolidation was that of NARF Alameda and Naval Supply Center (NSC) Oakland, which occurred in October 1979. Prior to that consolidation, Grant [Ref. 2] attempted to quantify the production delay costs caused by not having repair parts immediately obtainable when needed by researching NARF Alameda procedures and records. Although some costs, such as cannibalizations, had avenues for documentation, others did not and he was unable to develop a firm relationship between delivery times and delay costs. In the preparation of his thesis Grant conducted interviews at NARF Alameda and many shared one common view. Overall availability is much more important than the rapid delivery of less than all the parts required to repair a component.

It is really the slowest delivery which sets the pace of the repair action and should be used to determine production delay costs caused by the lack of repair parts. For example, if four parts were required and three were delivered in twenty minutes and the fourth was not delivered until two

weeks later, the component (barring cannibalization) would spend two weeks awaiting repair parts.

In a second thesis Davidson [Ref. 3] conducted an analysis of three direct delivery models which were initially proposed by McMasters [Ref. 4]. These models were based on a single customer (such as the jet engine repair line at NARF Alameda) and a single repair part which may need to be replaced and thus require requisitioning for each inducted component. The demand for this part was considered a Bernoulli trial with a fixed probability of demand (p) for each induction.

McMasters and Davidson attempted to minimize expected costs where total cost was the sum of transportation cost (a fixed charge per delivery) and delay costs (a fixed charge per component per unit of time delayed due to the lack of the repair part). The only delays considered were those caused by the transportation system (i.e. material availability was not considered) and the unit of time was defined as the time between component inductions on the repair line. Expected total costs were calculated, but due to analytic complexities of these models, closed form optimizations for the models were not possible. Instead, a parametric analysis was conducted for each of the three delivery plans.

Davidson showed that, although the plans considered differed significantly in form and emphasis, there was little difference in the optimal expected costs for each. She also showed that varying the delay cost per period (CD) had a much greater impact on the optimal total cost than varying the parameter p, although increasing p did increase cost.

This thesis will extend the work of References 3 and 4. It will consider systems with one or more customers, each no longer limited to one repair part per induction. Chapter 3 broadly summarizes the earlier models and then discusses additional assumptions needed to generalize these models. Finally it presents a new model for the deterministic case. Chapter 4 studies stochastic versions of the new model and Chapter 5 considers the impacts on this model of locating material at the customer's site. Chapter 6 presents a summary and conclusions.

III. SUMMARY OF MODELS CONSIDERED

The basic system being modeled is diagrammed in Figure 1

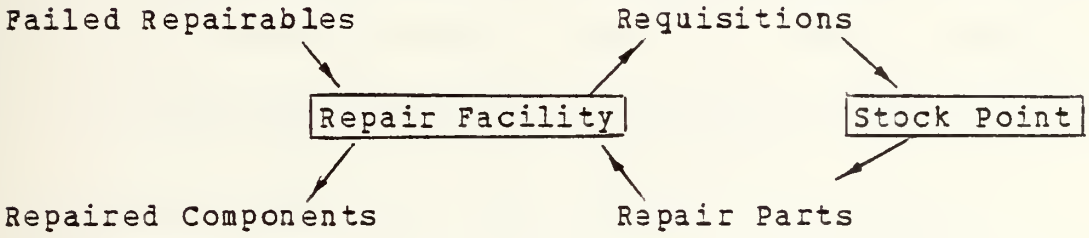


Figure 1: Customer-Stock Point Relationship

above. The industrial customers considered, such as production lines at a Naval Air Rework Facility, induct components for repair, troubleshoot each component, requisition any required repair parts, and, upon receipt of those parts, complete repair of the failed item. Earlier studies [Ref. 3 and 4] considered alternative transportation systems for delivering a given required repair part from the stock point to the customer and attempted to minimize the sum of expected transportation costs and expected customer delay costs. McMasters [Ref. 4] also addressed the establishment of an On-Site Inventory System (OSIS) at the customer's

location to expedite delivery and reduce customer delay costs. This study will address the OSIS in a later chapter.

McMasters proposed three basic local delivery options. These were:

1. Deliveries are made at the end of N periods if there has been at least one demand during that time frame.
2. Deliveries are made as soon as K issues accumulate.
3. A delivery is made in the (N-1)st period after the first demand following a delivery.

Initially this study will consider only Option 1. Davidson [Ref. 3] shows that for the single customer case all 3 models display nearly equal cost structures and recommended Option 1 as a quite reasonable strategy. Option 1 also seems best suited to non-industrial activities (such as ships in port) who must schedule workers based on parts availability. By knowing when deliveries are made, requisition status, and the ship's operating schedule, supervisors can estimate when technicians must work extra hours or when they can be given extra time off. Also, by knowing the delivery schedule, extraordinary action can be taken if system response will not satisfy a particularly critical need. Option 1 is also representative of the way many stock points currently operate their local delivery system.

This paper will modify the above model so that costs for a multi-customer (or multi-production line), multi-item inventory system can be considered. Cost structures of the model will be studied in the hope of determining rules for a cost minimizing delivery plan. The effect of locating supporting stocks at the site of the customer will also be studied.

A. ASSUMPTIONS MADE IN GENERALIZING FROM PREVIOUS MODELS

In generalizing to multi-item, multi-customer systems, some assumptions from the earlier model must be modified and some additional assumptions made. First, McMasters' model referred to a time period as "the time between component inductions on the production line". This is not convenient for the multi-production line environment where different customers may have different periods between inductions. Therefore a common denominator for time among all production lines or customers, the work day, is used in this study as the unit for time.

As with the earlier model, transportation costs will be considered as a fixed charge per shipment. In a multi-item inventory it might be more realistic to allocate charges by weight or volume, particularly if deliveries were constrained by one of those parameters. This was not done,

however, since local customers are being considered and it is felt that local deliveries are not usually capacity constrained. Moreover, by not being forced to specify specific item weight and cube, the model could remain more generally applicable.

Next it will be assumed that all requisitions are homogeneous within the issue and transportation system. This means requisitions are distinguished by requisition number and customer only, and not by priority, weight or cube, or item required. Although requisition quantity may be greater than one, issue of partial quantities is not considered. Finally, all requisitions are assumed to receive the same processing within the system.

By making these assumptions the multi-repair part local delivery problem becomes one of tracking multiple requisitions for each item under repair. Although this simplification does not allow for interdependent response times, such as might be expected when spares are driven to a not-in-stock position, it does allow for interdependence in the transportation system for the "ship every K issues" case.

It will be assumed that component inductions are made a fixed period apart. This period is a given parameter Y for each customer. Although it is usually determined by the

number of components scheduled for repair in the current calendar quarter, it can also be considered the maintenance time required for component repair given a maintenance resource allocation. As more components are required per quarter, Y will decrease and the shop supervisor will have to assign more production resources.

In the proposed model it is assumed the actual repair period is divided into three major phases. The first is the troubleshooting phase which is hypothesized to take one half the scheduled maintenance time, or Y/2 days. During this time the component is disassembled and all the parts which need to be replaced are determined.

Phase two of the repair process is the "obtain the repair parts" phase. It consists of ordering all required parts and waiting for their receipt. Since all requirements were determined in phase one, this phase takes essentially no maintenance effort. During this time, maintenance resources can be allocated to other jobs. Delay in receiving the required parts does incur costs in work in process inventory, maintenance test bench space occupied, and inefficiencies cause by moving maintenance personnel between jobs. For that reason, delay charges are assumed to be

assessed at a fixed rate (CD dollars) per component per day for the time spent awaiting repair parts.

The final phase of the repair process is the actual repair of the component. This includes replacing the failed parts, component reassembly, and final test. This last phase is allocated 50% of the maintenance effort, or Y/2 days.

It may be somewhat confusing that Y does not equal the sum of the three phases of the repair cycle. This is because more than one component can be in process at any one time, and should be if a component is awaiting repair parts. Y is the time between inductions, the average time between repair completions, and, in this model, the time between submission of requisition batches. The average component turn around time is the sum of the time spent in each of the repair phases, or Y plus average delay time.

The above three-phase repair process assumes that all the parts required can be determined in phase one and ignores any parts broken or discovered defective during reassembly. This is considered realistic since the rework facility has typically been repairing the component in question for a long time and these last minute demands can often be anticipated.

By assessing delay cost at a constant rate (CD) until all parts are received, any benefits of receiving some but not all repair parts are ignored. This does suppress any benefits from cannibalization, but cannibalization costs can be high and the above does seem the most fair way to levy delay costs.

B. DERIVING THE EXPECTED COST FUNCTION FOR THE IMMEDIATE ISSUE CASE

Consider the single customer case where the system has a transportation cost of CT dollars per delivery, and the customer has a delay cost rate of CD dollars per day per component and a scheduled induction period of Y days. The decision variable for the system is N, the periodicity, in days, of deliveries. The objective will be to minimize the average daily total cost where

$$\begin{matrix}\text{Average Total} \\ \text{Daily Cost}\end{matrix} = \begin{matrix}\text{Average Transportation} \\ \text{Cost Per Day}\end{matrix} + \begin{matrix}\text{Average Delay} \\ \text{Cost Per Day}\end{matrix}$$

or

$$ADC(N) = TC + DC .$$

To derive the average total daily cost, the process must be examined a little more closely. Consider first the single customer case. As long as N, the number of days between deliveries, and Y, the days between inductions of a component for repair, are rational, this will be a renewal

process. If rational, uY=vN for some integers u and v and the system will cycle every u inductions or v deliveries. To determine long run time-average costs, costs will only have to be averaged over a cycle. In the case of deliveries which cost CT dollars each, the total cost for the v deliveries of the cycle would be vCT. Since deliveries are N days apart, the total length of the cycle is vN days. Dividing the total delivery cost per cycle by the days per cycle, average daily transportation cost becomes

$$TC = \frac{v}{v}\frac{CT}{N} = \frac{CT}{N} \quad .$$

Delay costs are a little bit more complex for they are a function of both N and Y. In all, three different parameter conditions can be considered. First consider delay costs when N is less than Y. This implies deliveries are more frequent than inductions on the one production line considered. Although this may seem unrealistic in the single customer case since some deliveries would consist of no requisitions, it could easily arise when multiple customers at a single location or on a single local delivery route are considered. In any case, Figure 2a shows the time until the next delivery for a delivery schedule with N equal to 4 days. Superimposed on the x-axis and marked with triangles are the times when the requisition submissions would take

place if the induction periodicity, Y, equalled 4.5 days and the first delivery and order were concurrent. As can be seen from the figure, the delay for the first and ninth inductions would be the same and thus as long as N and Y remain constant, the length of component delay would cycle every eight inductions. Shown in the bottom graph of Figure 2 (Figure 2b) is the delay in days for each component. Note that if the initial delivery were a bit later it would increase the delay time for each of the seven subsequent induction in the cycle. Thus when calculating average component delay, this phase factor, call it q, based on initial conditions, should be added. However, it should be obvious that any optimal delivery plan should have initial conditions adjusted so that this q would be equal to zero. For this reason q will be assumed zero for the rest of this study.

Appendix A assumes both N and Y rational and solves for the values of u and v mentioned in the above renewal process argument. It derives component delay as a recursion relation and shows that the number of inductions in the cycle is N/L, where L is the largest real number common to both N and Y. L is defined such that Y/L and N/L are both integers, integers which are actually the u and v which were referred

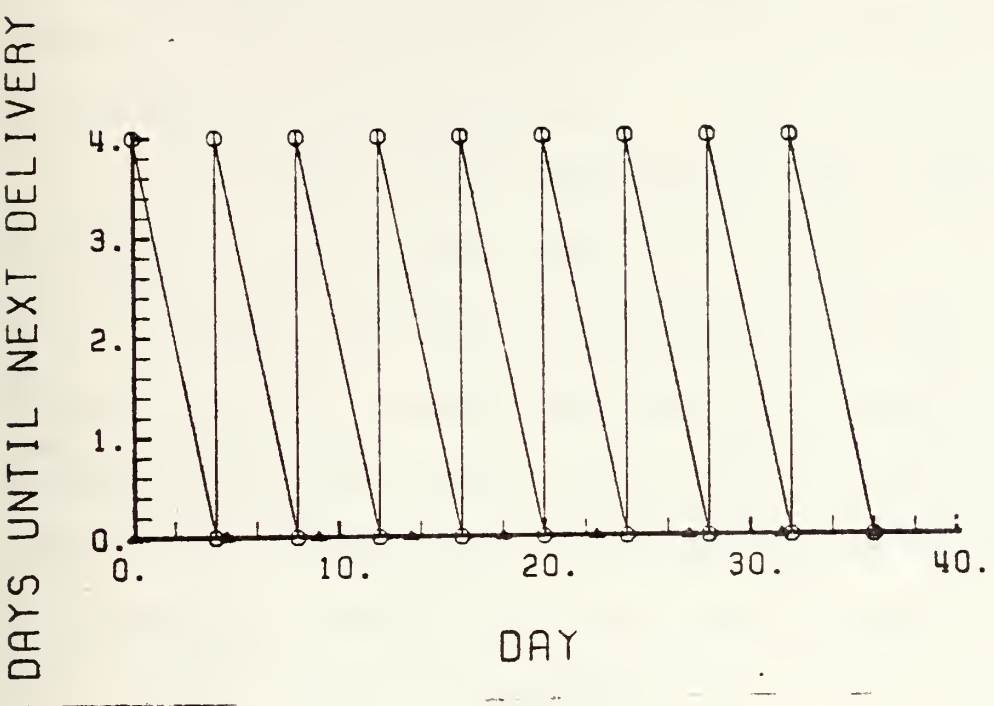


FIGURE 2A

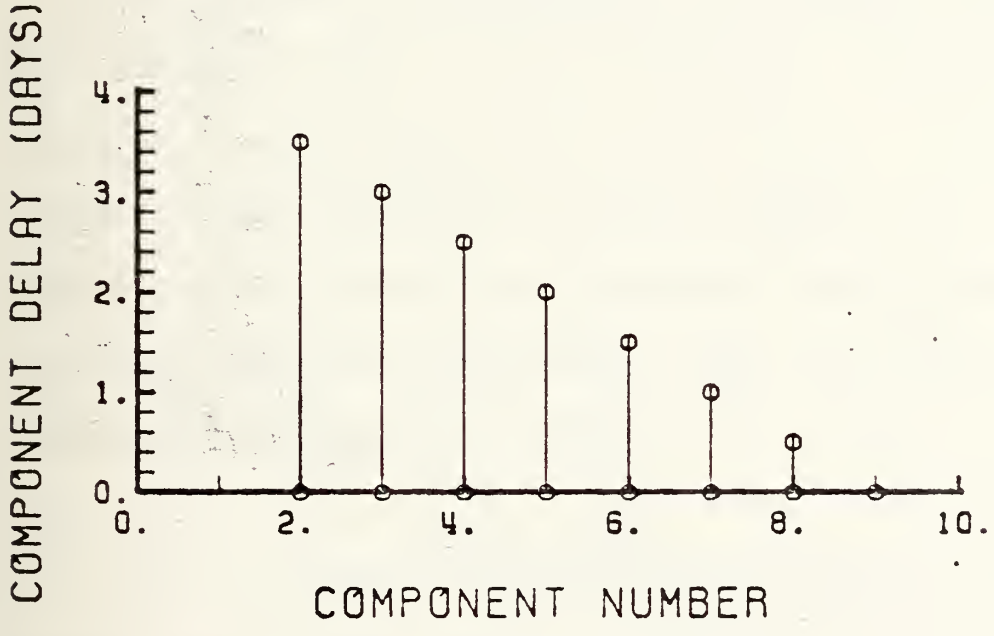


FIGURE 2B

Figure 2: Delivery Schedule- N less than Y

to earlier. The appendix then goes on to show that component delay accepts N/L evenly stepped discrete values and the mean of these values is

Average Delay per Component = ((N - L)/ 2) ,

yielding an average delay cost of

Average Delay Cost per Component = CD ((N - L)/ 2) .

Since this is the average delay cost per component and u components were inducted per cycle, the total delay cost per cycle would be uCD(N-L)/2. The length of the cycle would be uY days so the average daily delay cost becomes

$$DC = \frac{CD}{Y} \frac{(N-L)}{2} \quad . \qquad (3.1)$$

Next consider the case where N=Y, or where deliveries and inductions have the same periodicity. In this case all components would experience the same delay. As long as deliveries and orders were perfectly phased (i.e. initial conditions were right), each component would experience zero delay and hence zero delay cost. Note that this zero delay cost would be provided by equation (3.1) since L would be equal to N.

Lastly, consider the case where N is greater than Y. A special case of this condition is N=iY for some integer i greater than 1. Under this condition all the deliveries will still be at the same point in each repair cycle but now

more than one component will be awaiting repair parts. Assuming cost minimizing initial conditions, one component would experience no delay. Since i components would have been inducted since the last delivery, (i-1) components must have been waiting repair parts the last Y-day induction period, (i-2) the induction period before that, and so on. Thus the total component delay per shipment would be

$$Y((i-1)+(i-2)+\ldots+1+0), \quad \text{or}$$

$$Y\left(i\left(\frac{i-1}{2}\right)\right) .$$

Since there were v shipments per renewal cycle and N days between shipments, the average daily delay cost becomes

$$DC = \frac{1}{vN} \cdot v \cdot CD \cdot Yi\left(\frac{i-1}{2}\right) = \frac{CD}{N} \; Yi\left(\frac{i-1}{2}\right) .$$

But N=iY so

$$DC = CD\left(\frac{i-1}{2}\right) . \qquad (3.2)$$

This is the same deterministic delay cost equation as was developed by McMasters. Note that if i=1, delay costs are zero as was predicted earlier.

Equation (3.2) is also a degenerate form of equation (3.1) for the special case N=iY. Since N=iY, L must assume the value Y as long as i is integer. Using this fact, equation (3.1) becomes

$$DC = \frac{CD}{Y}\left(\frac{N - Y}{2}\right) = \frac{CD}{Y}\left(\frac{iY - Y}{2}\right) = CD\left(\frac{i - 1}{2}\right) .$$

Next consider the general case where N is greater than Y, or deliveries are less frequent than inductions. Although there may be more than one component awaiting repair parts at any one time, steady state average daily delay costs can still be obtained. Approaching the problem in a method similar to the N less than Y case, Figure 3a is a graph of the delivery schedule for N equals 5 and Y equals 3 days. The requisition times for a customer are marked as triangles on the abscissa. Note that in the case illustrated delay times within the cycle are not monotone decreasing as they were in the previous case (Figure 2), but delays are still in multiples of L. As the derivations in Appendix A still hold, average delay cost reduces to equation (3.1) again, or

$$DC = \frac{CD}{Y}\left(\frac{N - L}{2}\right) .$$

Combining the transportation cost and delay cost terms, the overall single customer daily cost function becomes

$$ADC(N) = \frac{CT}{N} + \frac{CD}{Y}\left(\frac{N - L}{2}\right) . \qquad (3.3)$$

where L is the largest number such that Y/L and N/L are integers.

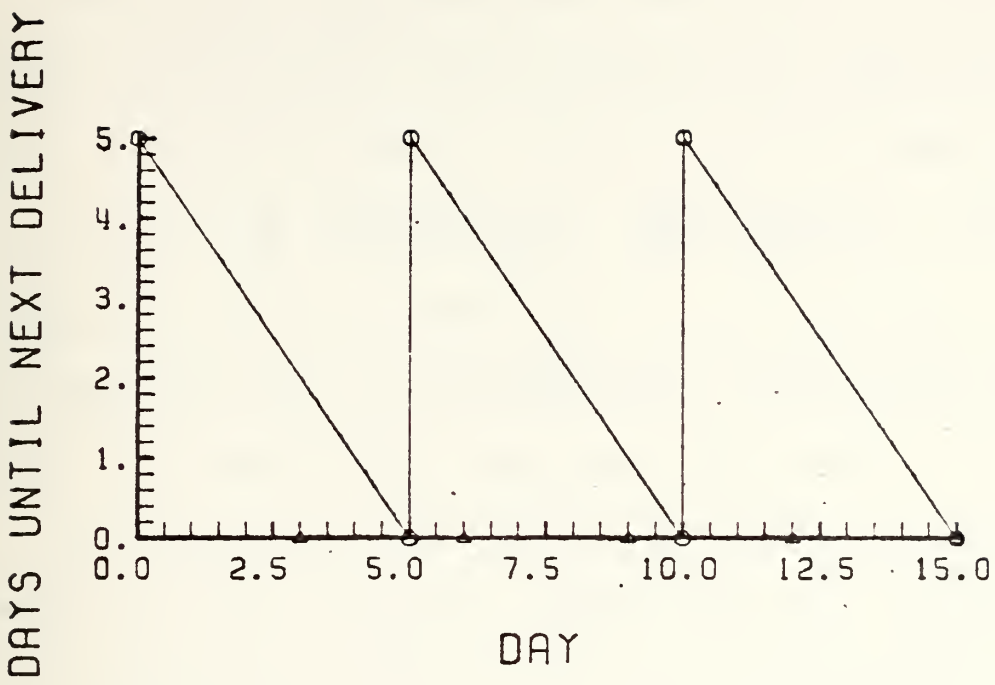


FIGURE 3A

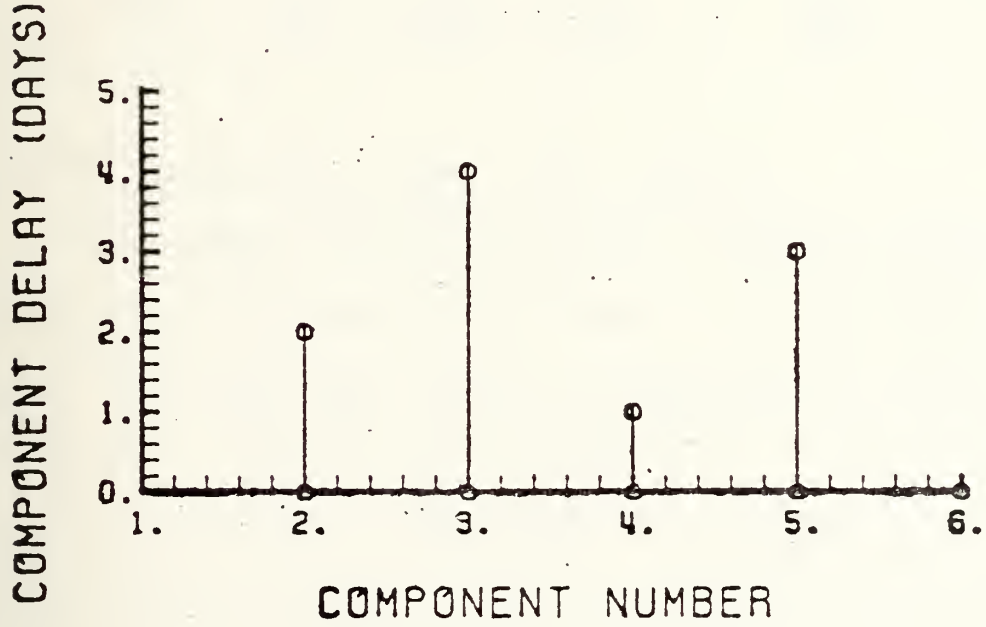


FIGURE 3B

Figure 3: Delivery Schedule- N greater than Y

For the two customers case where customer parameters are CD1, Y1, D1, and CD2, Y2, D2 respectively, delay costs for the customers are summed to obtain the system delay cost or

$$ADC(N) = \frac{CT}{N} + \frac{CD1}{Y1} \cdot \left(\frac{N - L1}{2}\right) + \frac{CD2}{Y2} \cdot \left(\frac{N - L2}{2}\right) \quad (3.4)$$

Equation (3.4) does assume that N, Y1, and Y2 are rational, or that deliveries can be phased such that at one point in time both customers can experience zero delay.

By ignoring the L terms, an upper bound approximation can be obtained for (3.4). Generalizing this approximation to n customers the average total daily cost can be expressed as

$$ADC(N) = \frac{CT}{N} + \frac{N}{2} \cdot \left(\frac{CD1}{Y1} + \frac{CD2}{Y2} + \ldots + \frac{CDn}{Yn}\right) \quad (3.5)$$

C. OPTIMIZING THE AVERAGE COST FUNCTION

Even though the cost expression given by equation (3.3) is only for the deterministic case, it is not easily minimized. The term which cause the difficulty in optimization involves L, which is not continuous in N. With that being the case, one way to "optimize" the function is to compute costs for the various values of N which are of interest and select, as optimal, that N which gives minimum plotted cost. Before doing this, however, it is possible to get an upper

bound on costs by deleting the L term from the cost equation. The resulting approximation is continuous in N and can be minimized using the calculus. Using the more general n customer case or equation (3.5) and solving for the first order conditions for minimization,

$$\frac{dADC}{dN} = -\frac{CT}{N} + \frac{1}{2} \cdot \left(\frac{CD1}{Y1} + \frac{CD2}{Y2} + \ldots + \frac{CDn}{Yn} \right) = 0$$

$$\text{or} \quad N^2 = \frac{2\,CT}{\frac{CD1}{Y1} + \frac{CD2}{Y2} + \ldots + \frac{CDn}{Yn}}$$

$$\text{implying} \quad N^* = \sqrt{\frac{2\,CT}{\frac{CD1}{Y1} + \frac{CD2}{Y2} + \ldots + \frac{CDn}{Yn}}} \quad . \qquad (3.6)$$

Checking the second order conditions

$$\frac{d^2ADC}{dN^2} = \frac{CT}{N} \quad ,$$

which is greater than 0 for positive N and CT, and thus N* given by equation (3.6) minimizes (3.5).

Figure 4 investigates the shape of this bounding cost function for the single customer case with CD=100 dollars, CT=100 dollars, and Y=3 days. It shows the total average cost and its components, transportation cost and delay cost plotted for various N, the delivery periodicity. When n=1, equation (3.5) is similar to the Hadley and Whitin [Ref. 5] Deterministic Lot Inventory Model cost function. As a consequence, the square-root formula for N* resembles that of the economic order quantity.

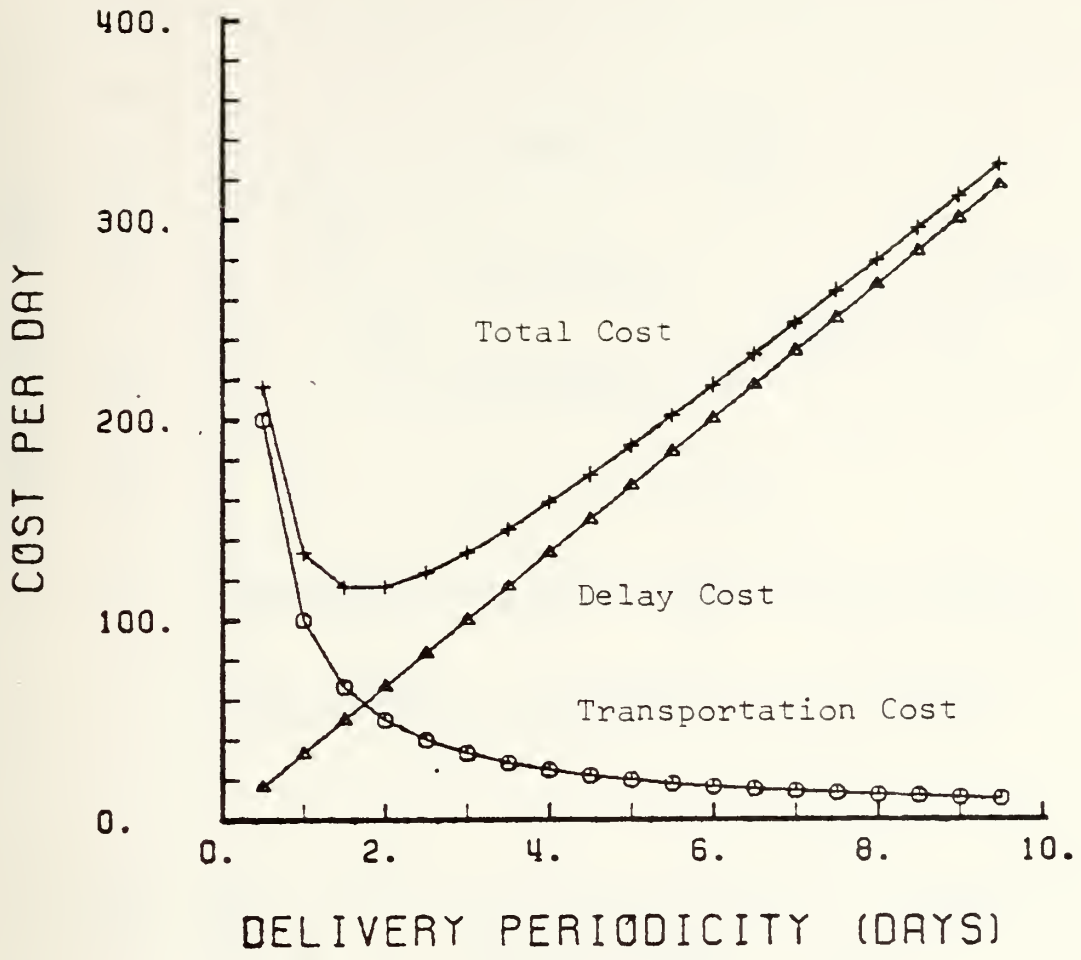
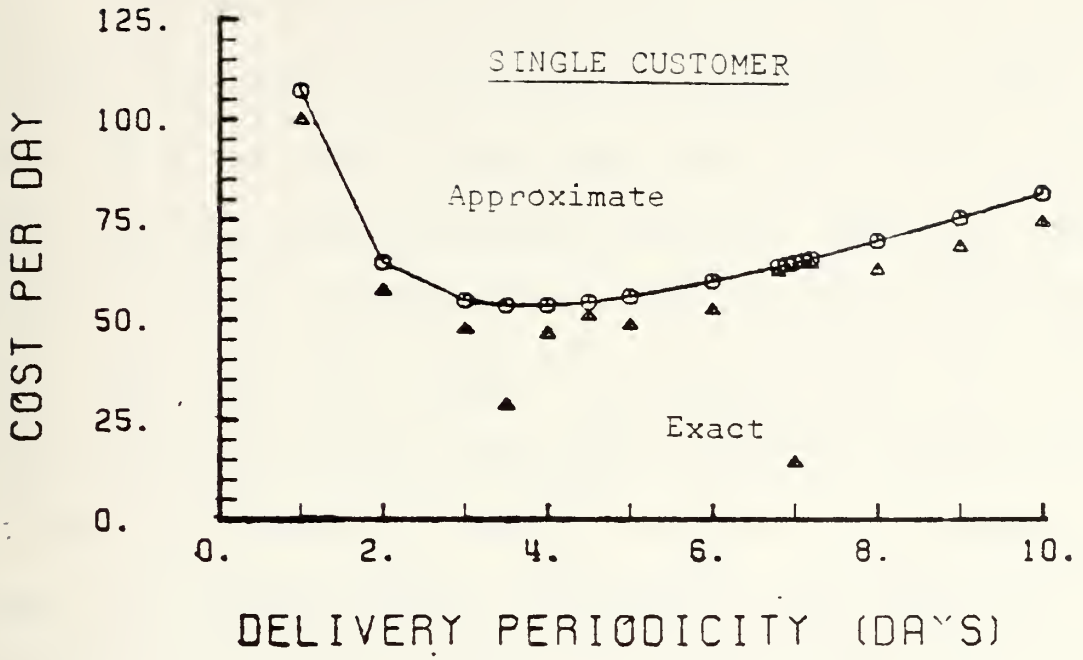


Figure 4: Approximate Cost Function Components

Figure 5 offers a comparison of the upper bound function and the exact cost functions in both the one and two customer cases. The top graph is that for a customer having CD=100 and Y=7 and a system CD=100. While the upper bound (approximate) curve is smooth and has a minimum near 3.74, the exact costs as derived from equation (3.3) and plotted as triangles, would not have a smooth curve. Although transportation costs are decreasing as N is increased, the



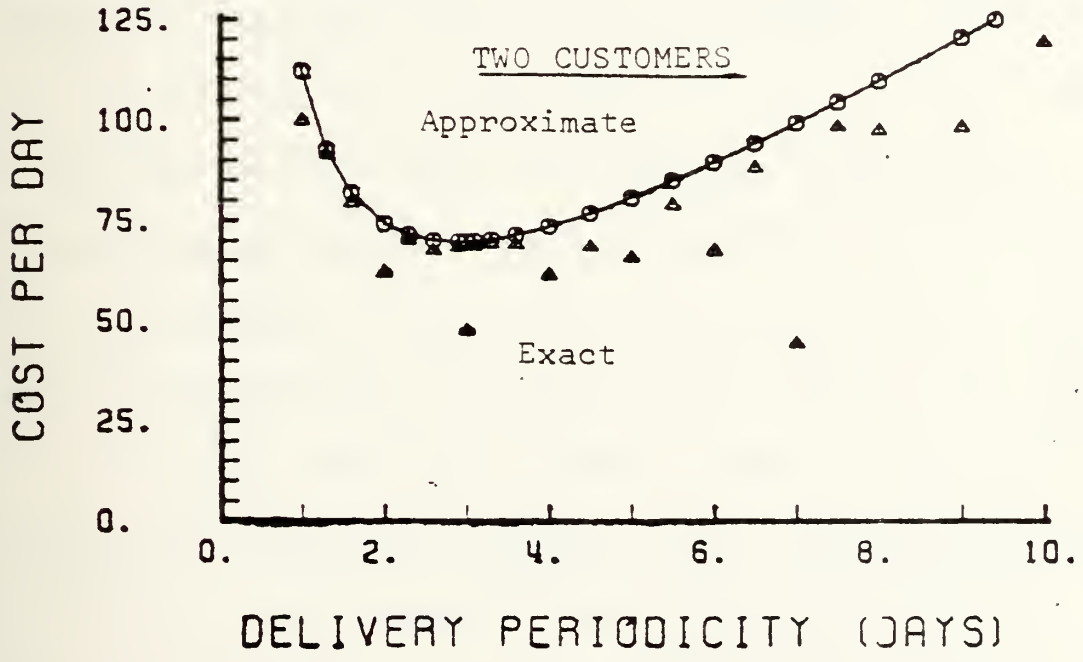


Figure 5: Approximate vs Exact Cost Function Comparison

delay cost term decreases and increases depending on relative values of L. At N=7 delay costs go to zero, giving a minimum total cost. Note however that varying N a small amount from this point gives costs which are near the upper bound function since L decreases sharply. Since the cost function is not continuous, lines should not be drawn between these exact cost points. Such lines would only encourage interpolation which could lead to invalid conclusions.

The lower graph in Figure 5 is for the two customers case. The second customer is assumed to have CD=30 and Y=3. Again the exact cost points, this time from equation (3.4), are plotted as triangles and again the minimum cost is not necessarily near the minimum of the approximate function. This time the exact cost points are the sum of three terms in the cost equation which act seemingly independently. Although transportation costs are decreasing monotonically as N is increased, the two delay cost terms increase and decrease depending on the values for L1 and L2. Note that at N=3 the delay cost for Customer 2 goes to zero since N=Y, but the delay costs for Customer 1 get a much smaller break. L1 at N=3 is 1 so Customer 1 delay costs are 2/3 of the upper bound amount. Also note that very slight changes from N=3 (such as N=3.001) give very small values for L1 and L2

and thus delay costs are very near the upper bound amount. Again the apparent minimum in Figure 5 appears at N=7. At this point Customer 1 experiences zero delay costs and Customer 2's delay cost is 6/7 of the worst case amount. This would not be the minimum except for the fact that the delay cost rate for Customer 1 (100) is significantly greater than that of Customer 2 (30). Although not plotted, again minor variations either side of N=7 yield delay costs and total costs near the worst case curve.

If N is greater than 7, N is greater than Y for both customers and both customers will experience some delay costs. Even though transportation costs are decreasing, it appears this decrease is less that the increase in delay costs and N=7 is the true minimum. Also note that in the two customers case the exact cost points more closely approximate the upper bound case in shape (although this approximation is still quite poor). As more customers are added to the system, more delay costs are added to the total cost expression. Thus each individual delay cost term is a smaller proportion of total costs and as long as the Y values are not the same, the upper bound approximation should improve.

Fortunately, values for N are not chosen in a continuous manner and deliveries are usually made every half day, day, or something like that. If Y and CD values for all customers are known exactly, the total average cost for each value of N can be calculated and that generating the minimum costs would be chosen as optimal. If only approximate values for Y are known, perhaps using the approximate or worst case function would be the best strategy.

IV. SIMULATION MODELS

As can be seen from the previous chapter, deriving delivery plans that minimize cost is difficult at best even when using a relatively simple deterministic model. When complicating factors such as stochastic demands, lead times, or induction periods are included, the mathematics quickly becomes extremely complex and is not easily analyzed through the use of the calculus. For this reason a simulation model of the system was written in the Simscript II.5 language. This program is an event step simulation and a listing of the basic program is included as Appendix B.

In an event step simulation, specific events are scheduled and executed at specific points in time. These events often lead to other events, which are then scheduled during execution. Figure 6 is a broad flowchart of the main events used to determine cost estimates for the system under study. The simulation allows using either the ship every N days or the ship every K requisitions delivery options. If the ship every K requisitions option is used, each time an issue is made the program determines if K requisitions have accumulated. If so, a delivery is scheduled. If the deliver

every N days option is used, the next delivery is schedule each time the delivery event (or subroutine) is executed. The simulation keeps track of who ordered each requisition so when it arrives it can be counted against the proper component under repair. It also keeps track of the progress of the components so that when repair is completed, delay costs can be assessed to the proper customer.

This simulation model accepts an arbitrary number of customers, each with its own delay cost rate, induction periodicity, and demand rate, as well as the system delivery cost and periodicity. An arbitrary issue delay or response time function can also be specified.

The simulation was used to generate points on the delivery frequency-average total cost curve, with random number generator seeds being reset for each set of parameters to reduce variability between simulations. The simulation was allowed to reach steady state before initializing counters for statistics and was then allowed to run for at least an additional 360 work days. The simulation was based on 24 hour work days and ignored the effects of customers not working on weekends and holidays.

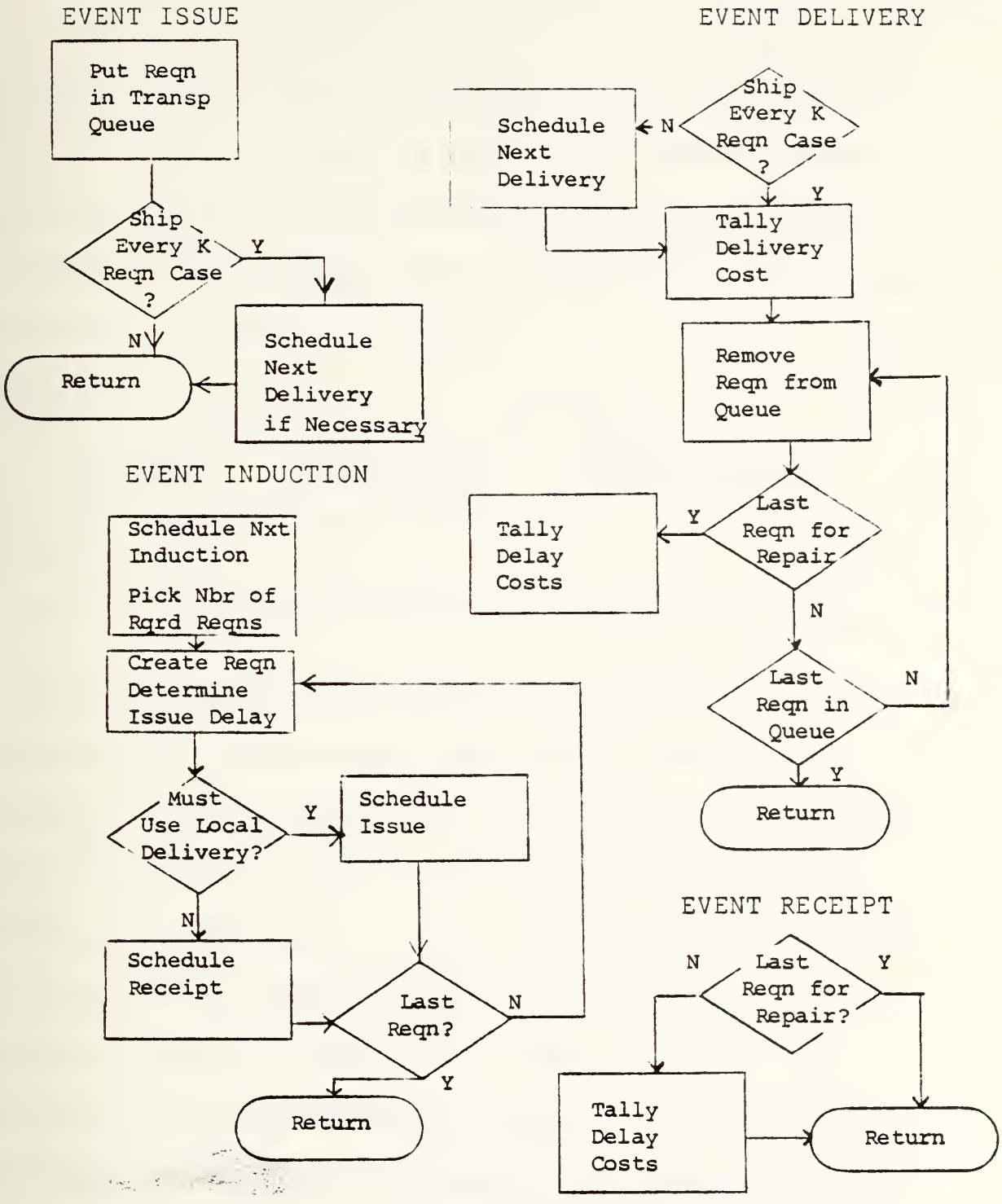


Figure 6: Simulation Model Flowchart

A. FIXED DEMAND RATE-RANDOM ISSUE DELAY MODEL

First consider the case where there is a random issue processing delay for each requisition submitted. Since much of the variation in delay as seen by the customer comes from the fact that all required material may not be available locally, non-local issues must be considered. This enlarged system is illustrated in Figure 7.

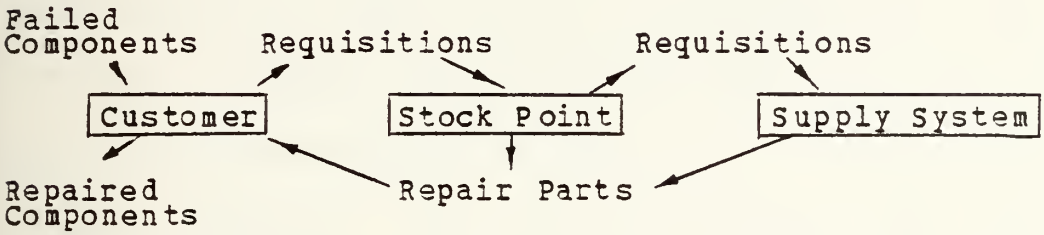


Figure 7: Enlarged Customer-Stock Point Relationship

To construct the issue delay function some assumptions on stock point effectiveness and system responsiveness were required. It was assumed that the local stock point would fill and deliver to the transportation officer 55% of the requisition submitted in 2.5 days, and another 5%, delayed for some unknown reason, would be filled and sent to local delivery uniformly throughout the next 4.5 days. After delivery to the transportation officer or local delivery, the requisitions would be delivered to the customer on the next scheduled delivery. The 2.5 day local issue delay value is based upon some requisition processing time at the

NARF plus standard issue processing at the supply center. The Uniform Material Movement and Issue Priority System (UMMIPS) standard for processing issue group two documents is 2 days. This can be improved through management attention and NSC Oakland has made it a policy to deliver all maintenance related material to NARF Alameda within one day. The 60% gross effectiveness at the local stock point may seem optimistic, but it should be realized that data collected by Hrabosky, Owen, and Popp [Ref. 6] showed that prior to consolidation, NSC Oakland was filling 36% of NARF Alameda referrals. This 36% plus whatever Naval Air Station Alameda was filling from stocks now carried by NSC Oakland may give the 60% effectiveness hypothesized.

It was also assumed that an additional 25% of the requested material would be available in the system and would be shipped by non-local means directly to the customer. It was assumed the material would be received somewhere between 7 and 15 days after requisitioning. It was assumed the remaining 15% of the items required would be out of stock and the backorder and/or procurement process would increase delivery time to the customer to somewhere uniformly distributed between 15 and 45 days. No repair parts

were assumed to have a leadtime in excess of 45 days (perhaps another optimistic assumption).

Because non-local deliveries are being introduced into the system and because only the local delivery system is being explicitly modeled, a further modification to the simulation was required. For parts issued non-locally, delivery time was included in the stated leadtime estimates while, for local issues, total requisition delay is the sum of issue processing time and the time to make the delivery. As a consequence, delay costs were divided into two components: 1) those caused by locally issued material and 2) those caused by non-local issues.

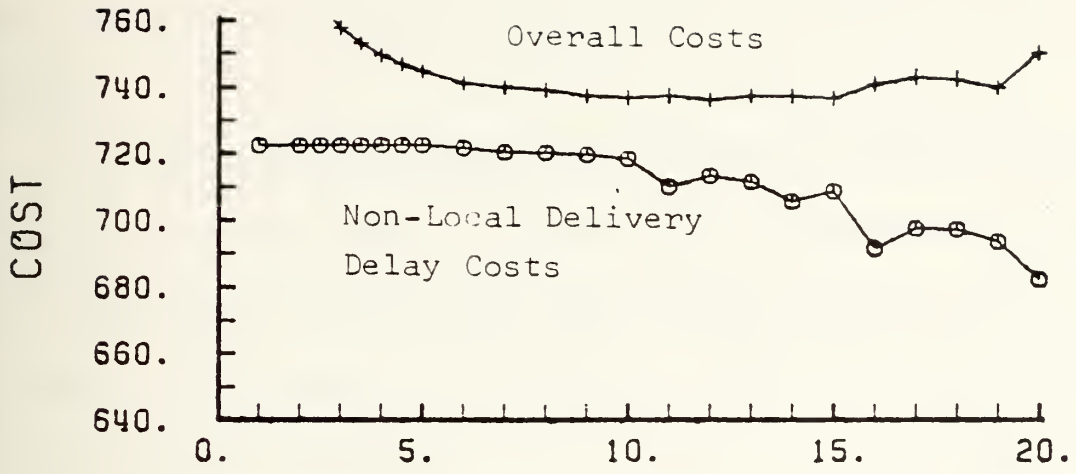
Because requisitions now have individual lead times, the number of requisitions submitted has now become a factor in delay costs. For this reason each production line supported has a new parameter, D, which is the number of requisitions submitted per component repaired.

A two-customer simulation was performed with parameter values CT=100 dollars, CD1=100 dollars, Y1=7 days, D1=14 requisitions per component, CD2=30 dollars, Y2=3 days, and D2=6 requisitions per component. Figure 8 shows the overall costs and delay costs contributed by local and non-local issues. This curve is being considered continuous even

though for the deterministic case it was not. With probabilistic variables in the model, the perfect phasing of requisition arrivals are no longer apparent in the model. Moreover, there are no longer the wild increases and decrease in costs noted in the simulation results. This continuity assumption will be made for all cost curves generated through simulation in this thesis.

Several interesting facts can be noted from Figure 8. First, local delivery delay costs take major jumps at N values of 11, 16, and 20, although these jumps are matched by decreases in non-local delay costs. These jumps are caused by the discontinuities in the issue delay probability distribution function and are believed to have no further significance.

Next, the delay costs have driven total costs much higher than in the previous chapter. Although response time for local issues has been increased 2.5 days, most of the delay costs are now coming from non-local issues. The non-local delay costs dominate the total delay costs for delivery schedules of 10 days or less resulting in a much flatter total cost curve than before. It is only when locally delivered material begins to arrive after non-local issues that total delivery costs begin to climb. It is likely that



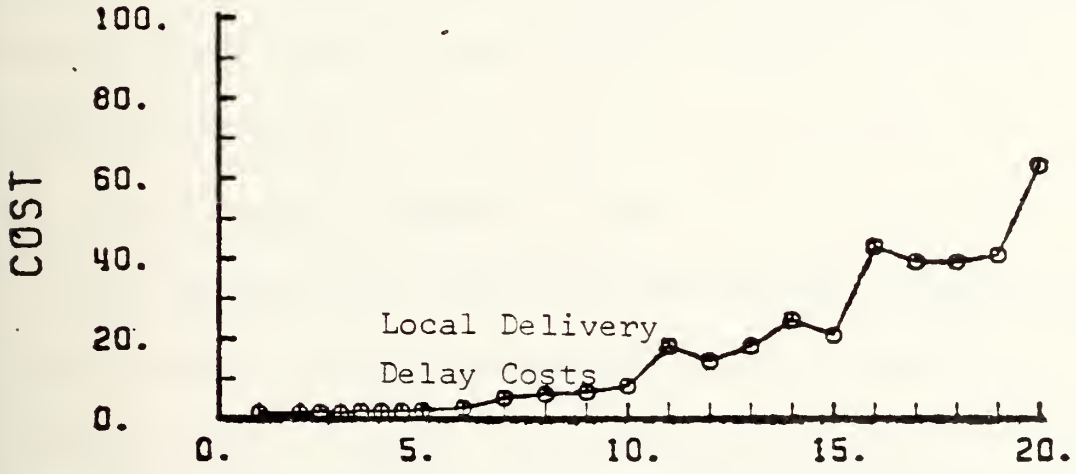


Figure 8: Fixed Demand Random Issue Delay Case

the local stock point would find such a schedule to be "undesirable" and would schedule deliveries more frequent than is optimal. However, because the cost curve is so flat the stock point would not be noticeably affecting the total cost.

It should also be noted that component delay is now a function of the number of requisitions submitted per component because of the assumed gross effectiveness values. Figure 9 is a graph of Customer 1's (14 repair parts per component) and Customer 2's (6 repair parts per component) average component delays in days versus delivery periodicity. As might be expected, the components which require fewest parts have a greater sensitivity to delivery schedules because they are more likely to have all repair parts available locally.

B. RANDOM QUANTITY DEMANDED CASE

Next consider the case where the number of requisitions submitted per component is random and the issue delay function is still in effect. In this case the number of requisitions per component is described by a probability density function. The underlying cause of a requisition is a failed repair part which is currently installed in the component under repair. Earlier studies [Ref. 3 and 4] considered the

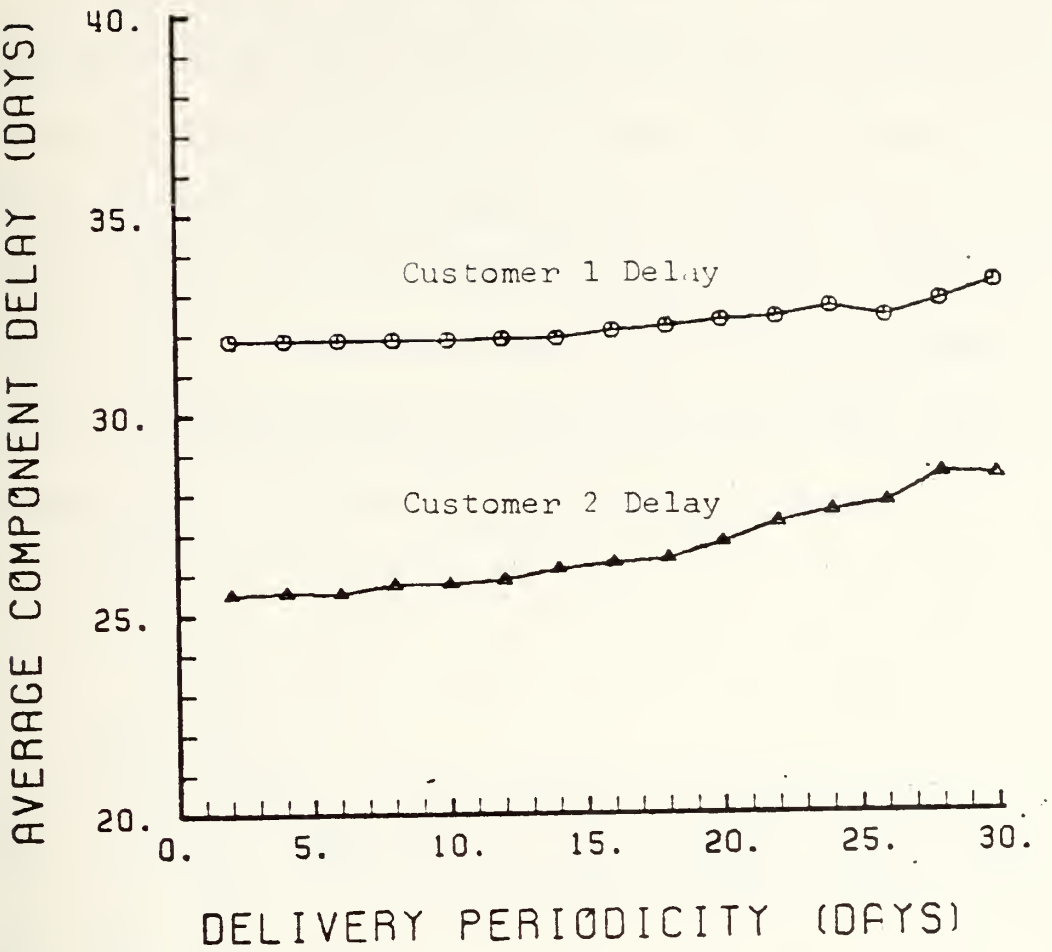


Figure 9: Customer Delay Comparison

determination of the need for a repair part to be a Bernoulli trial where the repair part would be replaced with probability p. For multiple like repair parts the sum of Bernoulli trials with a common p forms a binomial distribution. If parts are not alike, then the p's can be expected to be different and there is no nice distribution for arbitrary p.

For comparison purposes the same two customers from the previous section were modified so that each would have like components with probability of repair part failure of 0.5. By fixing the failure probability, the mean demand for each customer (D1 or D2) was used to calculate the number of Bernoulli trials or installed repair parts per component. Figure 10 compares the average cost curves under this modification with those of the previous section.

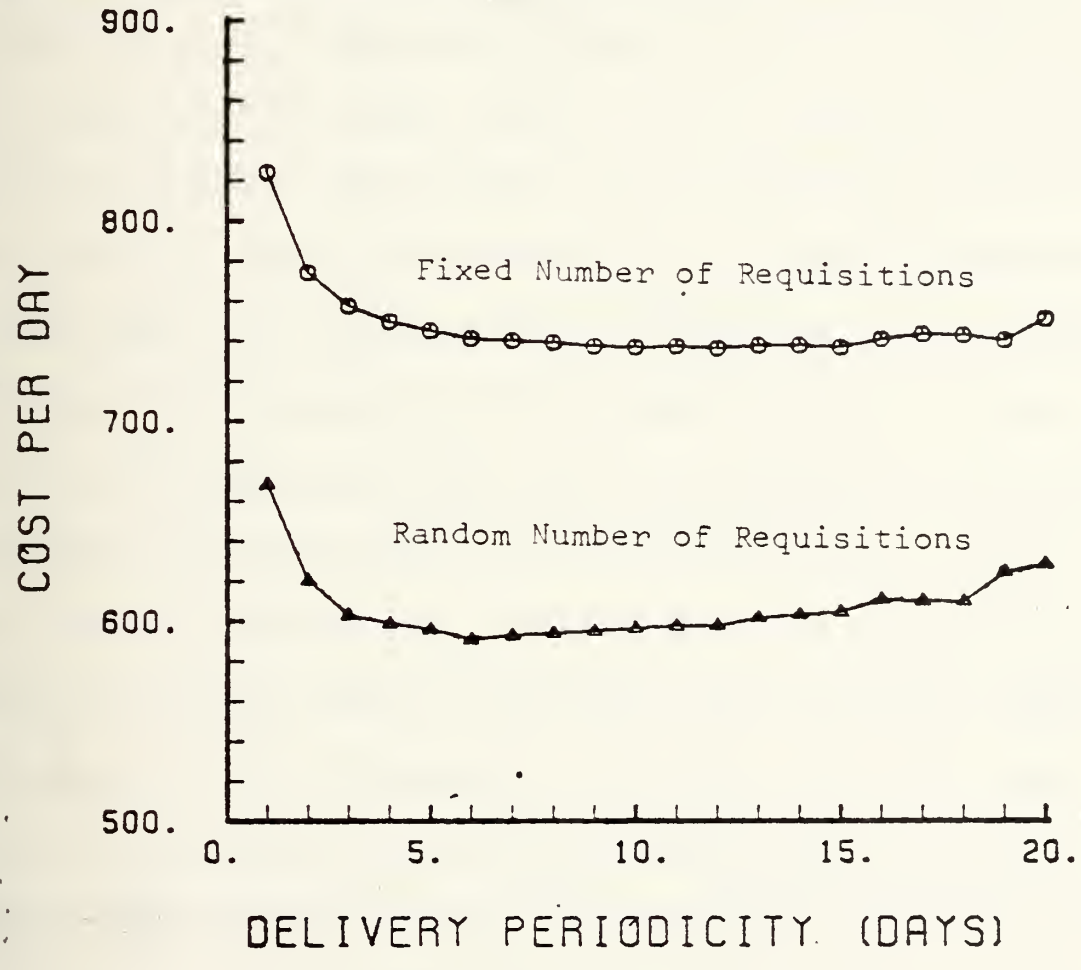


Figure 10: Random Demand-Fixed Demand Cost Comparison

As can be seen, costs have decreased about 20 percent from the earlier fixed demand case. This is because random demand has created a variation in the number of requisitions submitted. Since the binomial distribution is symmetric for p=0.5, this reduction is likely due to the increase in variance in the number of requisition per component. Apparently the benefit from one fewer requisition exceeds the cost of one additional requisition per component repaired. For a distribution with decreasing probabilities in the tails like the binomial, this seems logical. As the number of requisitions increases, each one has a lower probability of being the critical "last item received" which actually determines delay costs. On the other hand, as fewer requisitions are required, the probability of not ordering the item which would have determined delay (i.e. the probability of reducing delay cost) increases at an increasing rate. When N, the delivery periodicity, becomes large this argument can break down since being in the lower tail of the distribution becomes much less advantageous. In fact, if N were such that a local issue took as long to receive as a non-local issue, distribution variance should make no difference. It is doubtful a stock point would let local service degrade to this level, however.

A probability of repair part failure of 0.5 cannot always be assumed and the p value does have significant effect on distribution variance and shape. To investigate the effects of a varying p parameter, a simulation was run with 3 customers having identical parameters of CD=100, Y=7, and D=6 and only the binomial distribution p values were allowed to be different. Since it was previously shown that components which required more repair parts had higher delay costs, distribution means were made equal to 6 by varying n, the maximum number of repair parts that might need replacing, along with the parameter p. Customer 1 was assigned a p value of 0.1, Customer 2 a value of 0.5, and Customer 3 a value of 0.857. Figure 11 is a graph of the average component delay in days for each customer. Included in the graph are the delay costs experienced in the p=1.0 or deterministic demand case.

The customer with p equal 0.5 generally has the lowest delay, the one with p of 0.1 the second lowest, and the one with p of 0.857 the highest. As N gets large, the ranking is not so clear, however. As this happens, the "long lead time" non-locally issued requisitions actually begin arriving before the locally issued items. It appears this may be becoming a problem at N=20.

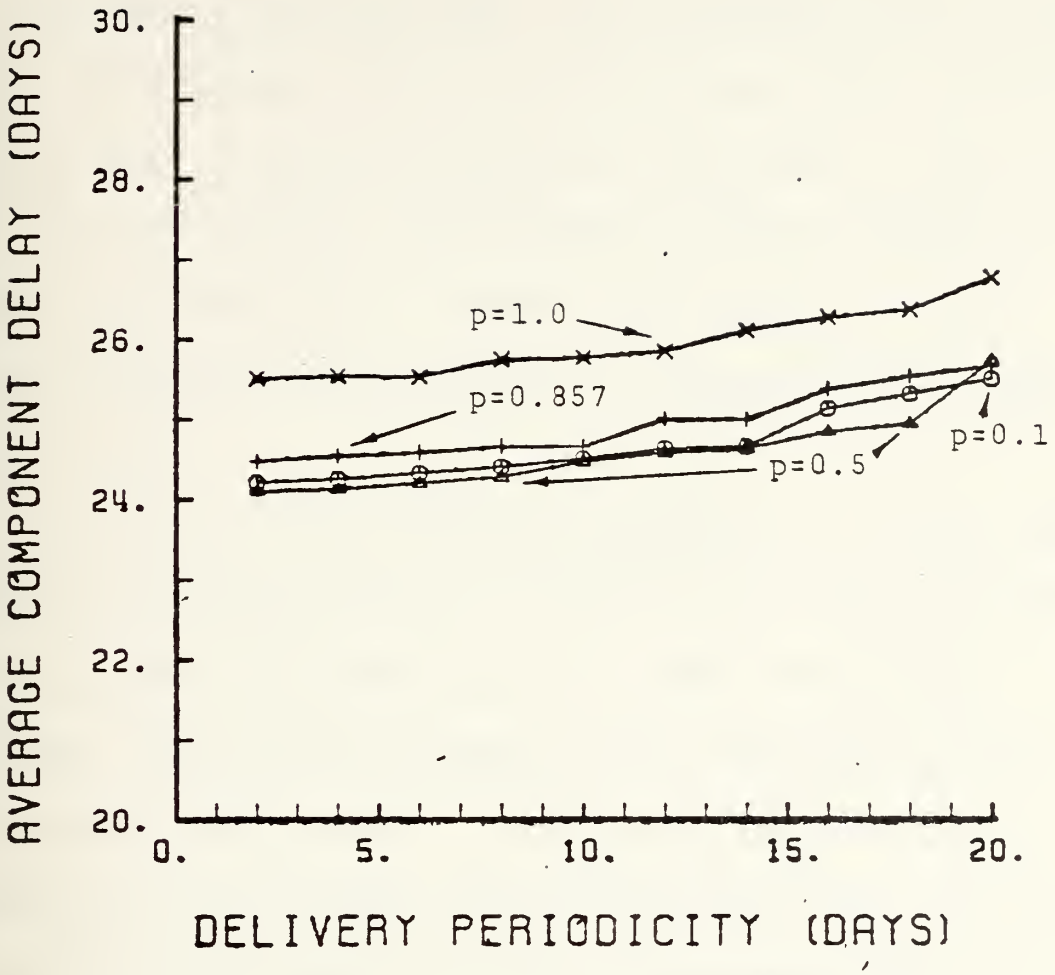


Figure 11: Delay per Component For Varying Failure Probability

For N less than 20 there are at least two forces at work, skewness and variance. With a p value of 0.5, the binomial distribution is symmetric about its mean (has zero skewness) so its mean is also the median. Thus equal numbers of components require more than the mean number of requisitions and fewer than the mean number. As discussed earlier in this section, a few more requisitions increase delay less than fewer requisitions reduce it, so there is a

net decrease in waiting time. As p decreases from 0.5 several things happen. First n, the maximum number of possible failed parts increases because the mean number of failures, np, was held constant. This means it will be possible for some of the components to need a large number of repair parts (perhaps n) and which, as a consequence, will dominate delay costs.

Variance is also affected by changing n. For the binomial distribution variance is $np(1-p)$, and since np is being held constant, the smaller p, the larger the variance becomes. Very small p values do have some traits which tend to increase delay costs. The distribution does become skewed so that the median is less that the mean. This means that the decreases from the mean are more frequent but less in magnitude. Deviations above the mean are infrequent but are quite expensive. These are the inductions which dominate costs as mentioned above.

Attempts were made to determine where the decrease costs from increased variance were overcome by the effects of higher distributional moments. Simulations were run with p values of 0.8, 0.67, 0.6, 0.4, 0.25, and 0.2. The corresponding n values were 7, 8, 9, 10, 15, 24, and 30, respectively. The differences in delay costs were so slight that

no strict ordering in costs could be obtained. All delay costs were below that observed in the deterministic case, however.

C. THE EFFECT OF VARIOUS FAILURE PROBABILITIES IN ONE COMPONENT

In the previous section it was assumed that all parts within a component had the same probability of replacement (or failure), p. It was also hypothesized that, barring effects from higher moments, an increase variance leads to slightly decreased delay costs. By examining the demand distribution for a component with two different p values, perhaps a statement can be made concerning delay cost estimates.

Assume, for example, a component had 2 classes of required repair parts, each with a different population (call them n1 and n2) and a different probability of failure (p1 and p2). Let the mean of the approximate distribution be equal to the sum of the two exact binomial distributions. Letting n1+n2=n, the aggregate demand parameter p can then be defined as

$$p = (n1p1 + n2p2)/n \quad .$$

To compare variances, the sum of the variances of the exact distributions should be compared to the variance of

the approximate distribution. For the exact distributions

$$\text{Var } D(\text{exact}) = n_1p_1(1-p_1) + n_2p_2(1-p_2)$$

$$= n_1p_1 - n_1p_1^2 + n_2p_2 - n_2p_2^2 \quad .$$

For the approximate case the variance is

$$\text{Var } D(\text{approx}) = np(1-p)$$

$$= (n_1p_1 + n_2p_2)\ (1-(n_1p_1 + n_2p_2)/n)$$

$$= n_1p_1 + n_2p_2 - (n_1p_1 + n_2p_2)^2/n \quad .$$

Next, set the difference between these two variances equal to a constant and attempt to determine the sign of that constant.

$$K = \text{Var } D(\text{approx}) - \text{Var } D(\text{exact})$$

$$K = n_1p_1 + n_2p_2 - (n_1p_1 + n_2p_2)^2/n - n_1p_1 - n_2p_2 + n_1p_1^2 + n_2p_2^2$$

$$K = -(n_1p_1 + n_2p_2)^2/n + n_1p_1^2 + n_2p_2^2$$

$$nK = -(n_1p_1 + n_2p_2)^2 + (n_1 + n_2)\ (n_1p_1^2 + n_2p_2^2)$$

$$= -n_1^2p_1^2 - 2n_1n_2p_1p_2 - n_2^2p_2^2 + n_1^2p_1^2 + n_1n_2p_1^2 + n_1n_2p_2^2 + n_2^2p_2^2$$

$$= n_1n_2\ (p_1^2 - 2p_1p_2 + p_2^2)$$

$$= n_1n_2\ (p_1 - p_2)^2 \quad .$$

For positive n_1 and n_2, nK and thus K must be positive, indicating the variance in the number of requisitions submitted in the approximate case must be greater than in the exact case. Although the above argument was for only two

binomial random variables, it can be generalized to an arbitrary number of values for p. Thus by using an appropriate binomial distribution, variance is being understated and, ignoring the effects of higher moments, delay costs are being overestimated.

D. EXAMINATION OF THE "SHIP EVERY K REQUISITIONS" PHILOSOPHY

As stated earlier, a study by Davidson [Ref. 3] showed little difference in the optimal costs for the local delivery options listed at the beginning of Chapter 3. To verify this in the multi-customer, multi-item inventory case, simulations were run to compare the "Ship every N days" strategy to the "Ship every K requisitions" philosophy.

In making comparisons between these plans, some sort of equivalency must be developed. Comparing a plan where K=10 with an N=2 may give one result when the system is delivering roughly 5 repair parts per day and quite another if on the average 50 repair parts per day are being shipped. For this reason it was decided to compare plans where the mean numbers of parts per delivery were approximately equal. Under the deliver-every-K-requisitions option, obviously the load is always K requisitions. For the deliver-every-N-days case, the mean delivery load is the average daily demand

times the proportion of requisitions shipped via local delivery (0.6 with the previously defined issue delay function) times the number of days between deliveries. Expressing this mathematically,

$$\text{Average Delivery Load} = \text{Average Daily Demand} \cdot (0.6) \cdot (N) \quad . \quad (4.1)$$

For each customer the average daily demand would be the average number of requisitions per repair divided by the period between repairs. Summing this for average daily demand for the two-customers case,

$$\text{Average Daily Demand} = \frac{n1p1}{Y1} + \frac{n2p2}{Y2} \quad . \quad (4.2)$$

Combining equation (4.1) and equation (4.2),

$$\text{Average Delivery Load} = (0.6) \cdot (N) \cdot \left(\frac{n1p1}{Y1} + \frac{n2p2}{Y2} \right) \quad . \quad (4.3)$$

Costs for delivery plans with equivalent average load values can now be compared.

Two-customers simulations were run with parameters p1=0.1, p2=0.1, n1=175, n2=75, Y1=7, Y2=3, CT=100, CD1=100, and CD2=30. Using equation (4.3) it can be seen the average delivery load should be 3N, or a plan with N=3 should be compared with a plan where K=9.

In Figure 12 average total costs were plotted against the average number of components per delivery for the two plans. These cost curves are nearly coincident and thus it

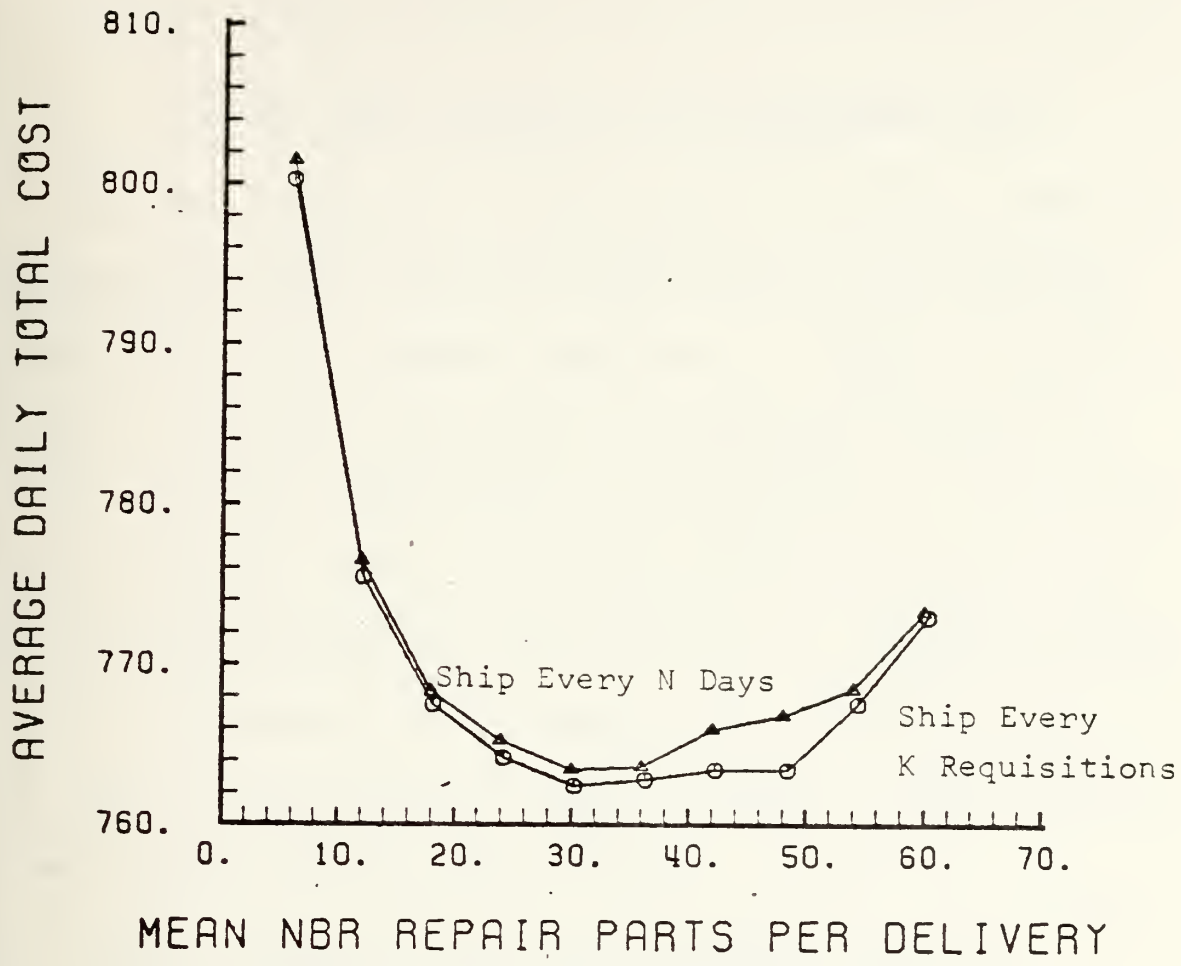


Figure 12: Cost Comparison for Two Delivery Strategies

appears the two delivery strategies are equivalent. It should be remembered, however, that it was assumed local delivery was not capacity constrained. If, in the delivery-every-N-days mode, material was not delivered because of a capacity constraint, then delay costs would be higher. For the deliver-every-K-requisition strategy such a problem could not exist since delivery capacity must be at least K requisitions for feasibility.

V. REPAIR PART STOCKAGE AT THE INDUSTRIAL SITE

The last problem to be considered is whether a separate warehouse facility should be maintained at the industrial repair facility to support operations. Although the model does not determine what should be stocked and where, the costs of various alternatives can be analyzed. For example, if establishing a local warehouse will decrease delay costs for one NARF production line much more than another, perhaps the local warehouse should concentrate on carrying stock for the line which derives the greater benefit. First, though, a more basic question must be asked.

A. FACTORS AFFECTING THE REMOTE WAREHOUSE DECISION

Many factors contribute to the decision of whether or not to establish a customer-sited warehouse. A review of some of these factors will place the delay cost problem in perspective.

First, the overhead of maintaining a separate, remote sited warehouse must be considered. Extra material and personnel are likely to be required. If automation in handling materials at the main warehouse has made it more efficient,

the increased costs of daily processing at the remote warehouse must be included.

Next, the source of material stored in the remote warehouse must be considered. If material is received primarily from off-base and is routed through a centralized receiving at the main supply center, handling the material at both the main center and the remote warehouse can incur significant extra cost. If, on the other hand, material represents components which have been made ready-for-issue by the industrial facility being supported and which are being returned to the system, significant savings can result by stocking the material at the remote site. This is especially true if the material is issued to another customer at the remote site, such as another NARF production line.

The speed of stock record take-up is another important factor, although costs are difficult to quantify. By avoid transshipment of repaired material to the main supply center, stock records can be updated sooner and, if requirements for the repaired component exist, the issue can be made more rapidly.

But perhaps the most obvious benefit of stockage of material at the customers' site is the decrease in requsition waiting time. By modifying the issue delay time

function introduced in the last chapter, delay cost impacts can be estimated.

B. SIMULATING THE SYSTEM WITH A LOCAL WAREHOUSE

In simulating a system with a local warehouse, only the issue delay function needs to be modified. It was assumed that if such a warehouse existed it would fill 40% of the demands submitted by the co-located industrial customers. This gross effectiveness is just a rough, perhaps pessimistic guess at what might be obtained by a standard, demand based, stocking policy. By making issues locally, material would not have to enter the supply center's local delivery system and, it was assumed, would be available to the customer in exactly one day. The issue delay function was modified accordingly and the simulation was run for four co-located customers using the policy "ship every N days". Four customers were chosen to provide a spread in customer parameter values. The number of demands per component for each customer were binomially distributed with p=0.1 and all customers were assigned a delay cost rate (CD) of 100. Y (the time between inductions) and D (number of requisitions per induction) were equal for each customer but were different for each of the four, being 18, 12, 6, and 3, respectively. These values of Y and D allowed each customer to

have an average demand rate of one requisition per day, yet provide a spread in the average number of requisitions per component. Figure 13 is a comparison of the local warehouse

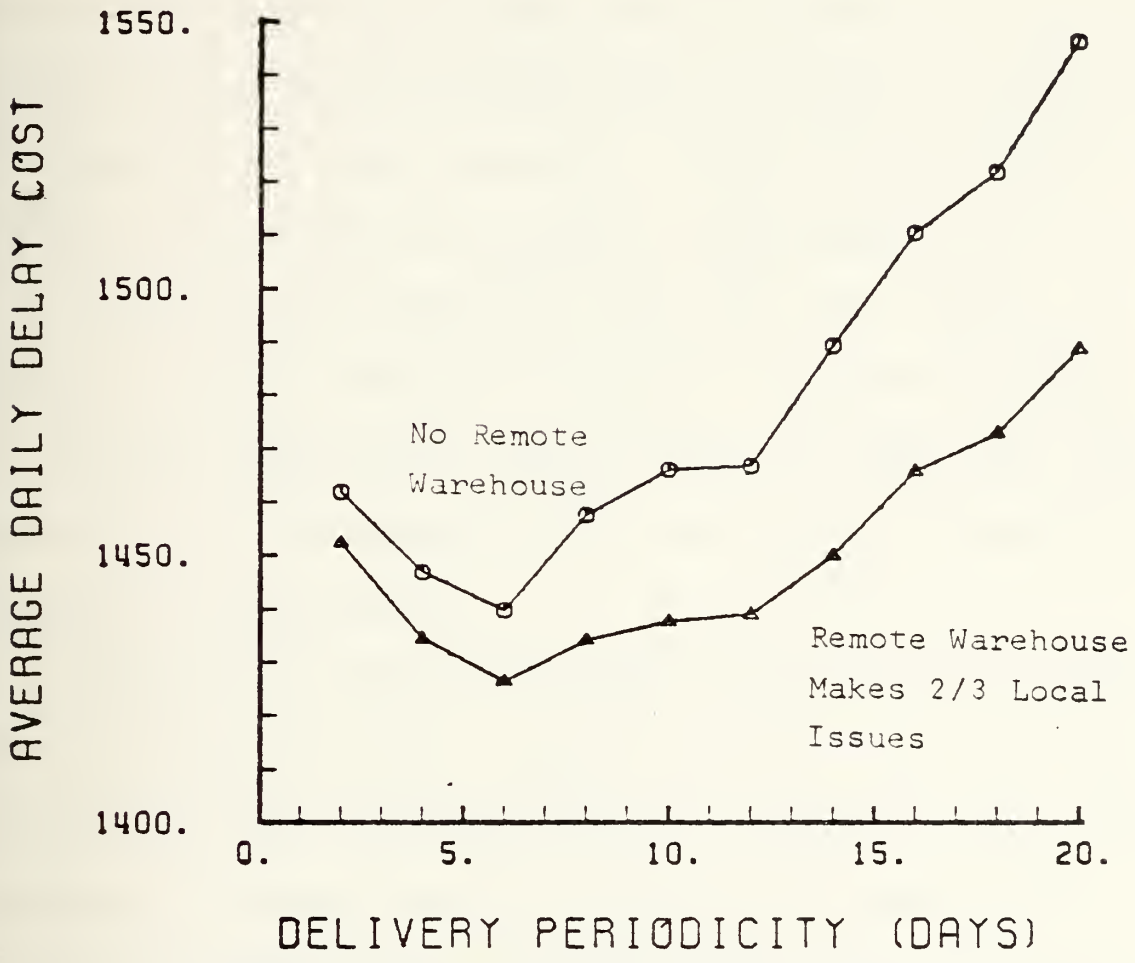


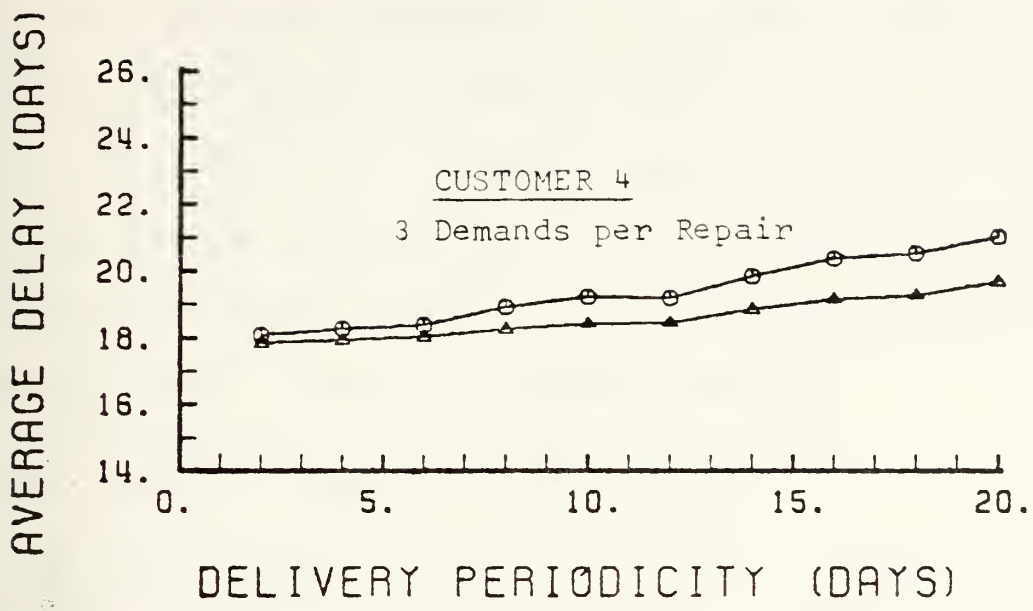
Figure 13: Local vs Non-Local Stocking

simulation versus the no local warehouse simulation. As can be seen, costs were roughly 1 to 4 percent lower for the local warehouse case and only 1 to 2 percent lower for values of N less than ten.

Next it could be asked which of the above four customers benefitted the most from the local warehouse. If Customer 4 (3 demands per repair on the average) showed a significant delay cost reduction, perhaps more of his material should be stocked in the local warehouse even at a cost of having less material for the other customers. Maybe material should be stocked so that all of his supply center issues should be made from the local warehouse while only a few issues are made locally for the other customers. Of course, stocking to a higher effectiveness usually requires higher and higher investment per incremental issue, and perhaps a cost-benefit analysis is appropriate.

Using the same simulation model as for the previous figure, individual customer average component delays were calculated for each warehousing plan. For both Customer 1 and Customer 2, average component delay was the same with or without the remote warehouse for all values of N between 2 and 20 days. Figure 14 shows the graphs of average component delay for the other two customers.

It appears Customer 4, the customer who on the average only required 3 repair parts per component repaired, would benefit most from a co-located warehouse. Customer 3 (6 demands per repair) would also benefit some, though it



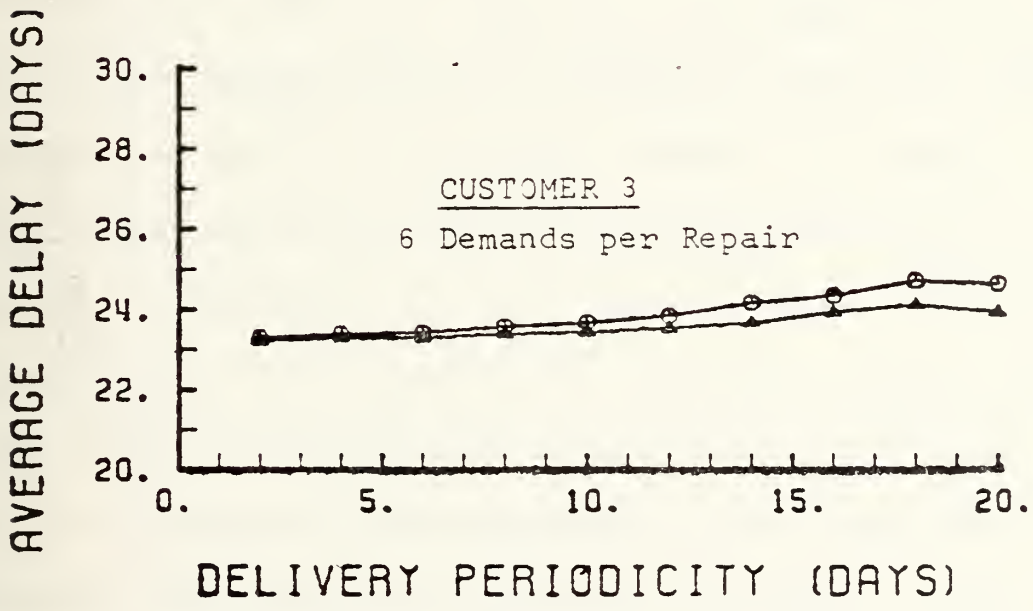


Figure 14: Customer Delay Cost Comparisons

appears as the average number of requisitions per component increase, the benefit derived from a local warehouse decreases. As might be expected, as deliveries become more frequent, local warehouse benefits also decrease. Thus given similar delay cost rates and demand rates, if a local warehouse has been established, delay costs can be reduced by targeting stocked material to the customer who require the fewest repair parts per component repaired.

C. THE EFFECTS OF IMPROVED EFFECTIVENESS

Throughout earlier analyses it has been assumed the supply center has been limited to 60% point of entry (POE) effectiveness. What would happen if, by studying past failure data and possibly making increases in range and depth, effectiveness could be increased? Assume, for example, the supply center could fill 75% of the NARF requisitions in 2.5 days and an additional 5% in the next five days. If the remaining 20% of the requisitions were split evenly between system issues (7 to 15 days from requisition date until receipt by customer) and backorders (15 to 45 days until receipt), a new issue delay function is defined. Using the same four customers as in the simulations used for Figure 13, cost curves were generated for this new issue delay function. The new curves are shown in Figure 15. The

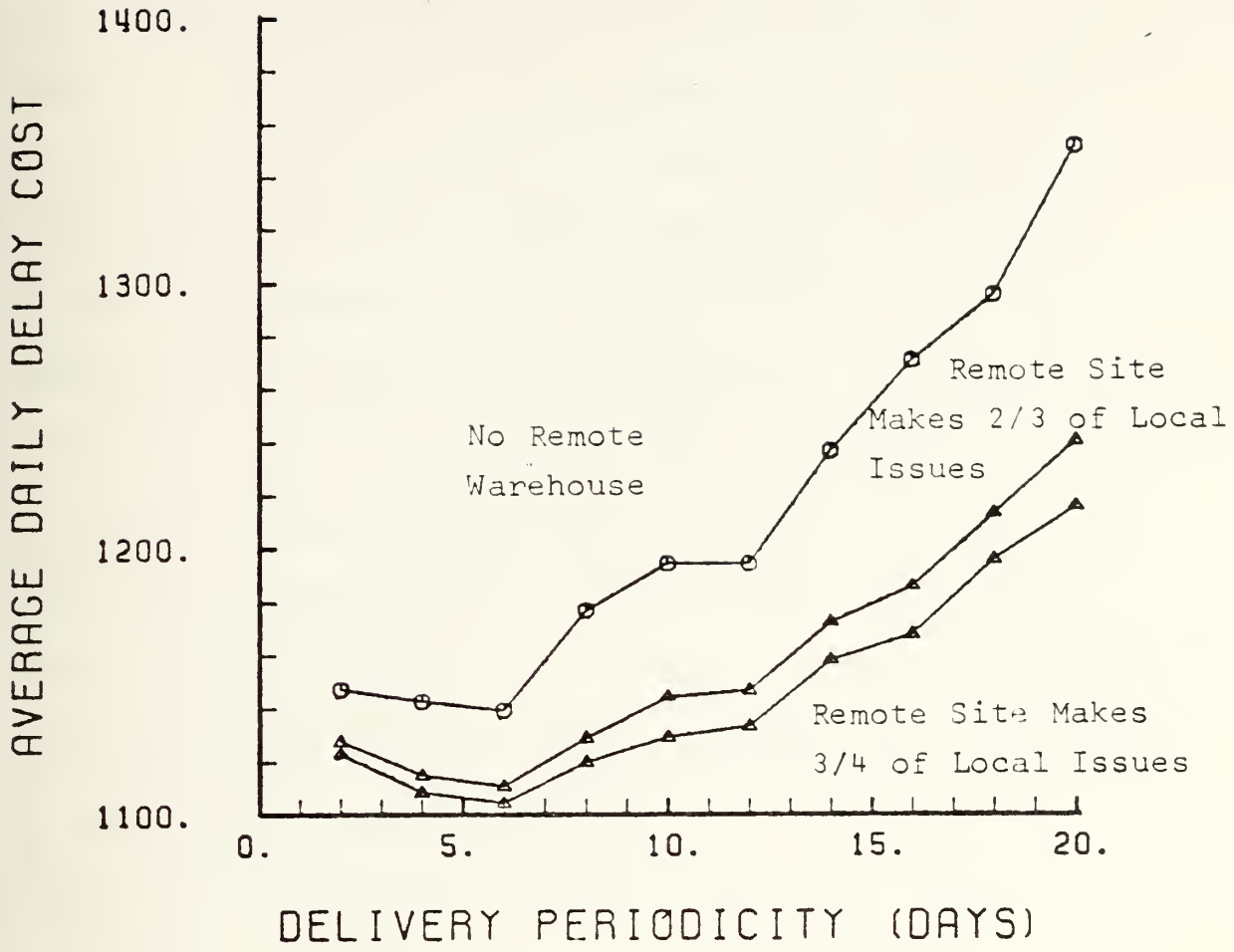


Figure 15: Local vs Non-Local Stocking-Enhanced Effectiveness

highest curve is for the case of no remote, customer-sited warehouse. The middle curve is for a remote warehouse which makes two thirds of the supply center's issue to the NARF, the same percentage as was considered earlier. The lowest curve represents a remote warehouse which is able to make 75% of the supply center's issues to the NARF.

As can be seen by comparing Figures 13 and 15, with the higher effectiveness total daily costs for N between 2 and 6

have dropped nearly 22% for the no local warehouse case. When it is assumed the local warehouse makes two thirds of the NSC's issues as before, the percent cost reduction is even slightly greater. Further improvement is possible, however, by assuming the local warehouse can make three quarters of the NSC's issues to the NARF. This is shown by the lowest curve in Figure 15. Since more issues are made locally, it is more likely all material is available and rapid local response can be converted into lower delay costs. Thus it appears that success feeds upon itself and those activities with the highest effectiveness can benefit the most from a remote warehouse.

VI. SUMMARY AND CONCLUSIONS

Unfortunately this paper was unable to find a simple solution or algorithm for the optimal delivery or siting of repair parts. When the local delivery problem is generalized to a multi-customer, multi-repair part inventory environment, the number of different parameters becomes significant and even in the relative simple deterministic case the cost function lacks continuity as well as convexity. Nevertheless, there is an upper bound function which can be optimized. This bounding function has a well defined minimum which is similar in form to the cost function in the Hadley and Whitin Deterministic Lot Inventory Model. It was also noted that the number of requisitions submitted per component repaired had no effect on costs.

In generalizing to the multi-item inventory, which allowed more than one requisition per component repaired, a key assumption was made concerning the assessment of delay costs. By allowing delay costs to accumulate at a constant rate until all ordered parts were received, much more emphasis was placed on requisitions with the slowest delivery

times. When a function which simulates system response times as well as local issue processing times was constructed, these slow requisitions were generally issues outside the local system and thus not a function of the local delivery schedule. As a consequence, optimal costs became very insensitive to delivery schedules. At the other end of the spectrum, delay costs became sensitive to the number of requisitions ordered per component, particularly when that number was small. Those components requiring few parts could more often have all requirements filled at the requisition point of entry (POE) and thus experience minimal delay. An increased POE effectiveness would also provide a similar decrease in delay cost.

Delay costs were also decreased when variability was allowed in the number of requisitions per component. Variance, though, was not the only distributional moment which affected delay costs, for costs also seemed to increase as the p decreased below 0.5 using the binomial demand distribution. More study should be conducted in this area.

Lastly, the warehousing of material at remote sites was considered. As modeled, a warehouse located at the customers' site had little impact unless exceedingly few parts were required per component. The model assumed only 40% of

all demands could be filled from the local warehouse and a greater effectiveness could give greater delay cost reductions. It appears high effectiveness and targeting material towards customers who require only a few repair parts per component is essential to deriving full benefits from remotely located warehouses.

With respect to customer response time the following conclusions can be made. First, this study shows that non-local deliveries and POE effectiveness are usually the limiting factors in delay costs. Although most issues for local customers will not reduce waiting time, many issues to non-local customers may be critical "last part required" and thus reduce system delay costs. This means that when an activity such as NSC Oakland invests in equipment which reduces response time, not only are delay costs reduced at local customers such as NARF Alameda, but there also may be reductions at other major customers such as Ship Repair Facility, Subic Bay, Philippines, or Ship Repair Facility, Yokosuka, Japan. Expeditious deliveries to fleet units located at the industrial site are important, since the lack of repair parts may be directly affecting fleet readiness.

Next, as might be expected, the more requisitions ordered, the greater the delay cost. Thus if many required

items are stocked as planned requirements, pre-expended bin, or in repair kits, the total number of requisitions submitted to the POE at the time of repair can be decreased, decreasing delay costs.

By their nature, delay costs are somewhat nebulous and the results of this study could be challenged on those grounds. Perhaps having some repair parts rapidly available would decrease delay costs. Perhaps an upper bound (or time standards) on supply response time is appropriate such that delay costs would only be assessed when this time is exceeded. Unfortunately, time standards are now dictated by the system rather than by individual repair processes. The Naval Avaiation Maintenance Plan (NAMP) says only that issue group one material must be delivered within an hour and issue group two and three in two hours and twenty four hours respectively, regardless of the repair process. Lastly, perhaps delay costs are not time dependent and only a fixed charge should be assessed if time standards are not met. More investigation on the nature of delay costs appears in order.

This study also assumed all requisitions were treated equally by the system. There were no issue priorities, premium transportation, or material expediters. Expediting

critical delay-causing requisitions would be a particularly effective way of decreasing delay costs in this model. This could ideally be done through computerized requisition submission, follow-up and monitoring programs. Only through good local requisition processing and expedited system support can industrial facilities keep the depot turnaround time to a minimum and operational availability at a maximum.

APPENDIX A

MATHEMATICAL PROOF OF AVERAGE COST IN THE DETERMINISTIC CASE

Consider the component delay problem as illustrated in Figure 2 of Chapter 3. As long as both N (the number of days between deliveries) and Y (the number of days between inductions or equivalently between requisition submissions) are constant and N/Y is a rational number, component delays are cyclic. Moreover, the average delay per component over the cycle can be calculated.

Theorem 1: If N/Y is a rational number and N and Y are constants, the values for delay cost will be cyclic over time.

Proof: The figure below shows a timeline of two component repairs where D(0) and D(1) are the delay times for two consecutive components.

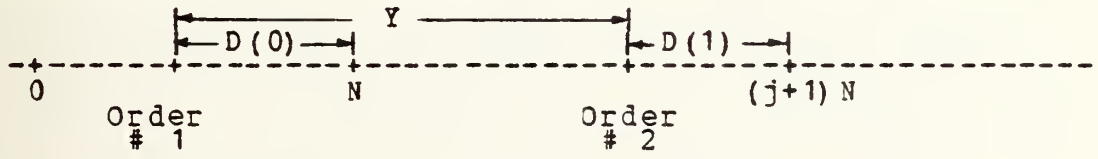


Figure 16: Repair Timeline

The key points to notice in the figure is that deliveries are an integer times N days apart, inductions are Y days apart, and delays are measured from an induction to the next

delivery. From the figure

$D(1) + Y = D(0) + jN$ for some integer j, or

$D(1) = D(0) + jN - Y$.

For the i th component inducted, this expression becomes

$D(i) = D(0) + jN - iY$,

where j is an integer and is chosen such that D(i) is the smallest positive number possible. But since N/Y is rational,

$$\frac{N}{Y} = \frac{u}{v} \quad \text{for some integers u and v.}$$

If this is the case, uY = vN, or D(k) = D(0) since j can be chosen to be v. Thus delay values are cyclic every u inductions and the cycle length is uY days.

Although as drawn it appears N is being restricted to a value less than Y, this is not necessarily the case. If j=1, as long as D(0) is less than Y (as it must be for some component whose last repair part is delivered at time N), the above and below arguments hold, although the figure may not be to scale. D(1) may accept values greater than Y.

Theorem 2: If delay costs are cyclic, the average delay over the cycle in days is

$$D = \frac{N - L}{2} + q$$

where L is the largest real number such that N/L and Y/L are

both integer and q is some constant between 0 and L.

Proof: Since delay costs are cyclic, let D(0) be an arbitrary delay observed in a cycle. Subtracting an constant q from each observed delay the recursion relation from Theorem 1 above becomes

$$D(1) - q = D(0) - q - Y + jN \quad \text{or}$$

$$D(1) - q = N \cdot \left(\frac{D(0) - q - Y + jN}{N} \right)$$

Since deliveries occur every N days and orders are filled immediately, the maximum delay will be no more than N days. If that is the case, the expression in parentheses must only assume values less than 1. If the calculated delay, D(1)-q, is greater than N, the required part would have been delivered with an earlier delivery N or some multiple of N days earlier. This leaves only the fractional part of the above expession in parentheses as delay. The expression can then be rewritten

$$D(1) - q = N \text{ Fractional Part } \left(\frac{D(0) - q - Y + jN}{N} \right)$$

Next divide both the numerator and the denominator of the fraction and both sides of the equation by L, where L is the largest real number such that N/L and Y/L are both integer. The expression then becomes

$$\frac{D(1) - q}{L} = N \text{ Fractional Part } \frac{\frac{D(0) - q}{L} - \frac{Y}{L} + \frac{jN}{L}}{N/L}$$

$$= \left(\frac{D(0) - q}{L} + \frac{jN}{L} - \frac{Y}{L}\right) \mod \frac{N}{L} \qquad (A.1)$$

where mod is the modulus function. Using this latest expression, first consider the term

$$\frac{jN}{L} - \frac{Y}{L} = c \quad .$$

Since j, N/L, and Y/L are all integer, this expression, call it c, must be an integer. Moreover, since this term appears within the parentheses, j can be adjusted so that c accepts values between 0 and N/L without affecting the equation.

Next define the general expression

$$\frac{D(i) - q}{L} \quad \text{as} \quad X(i) \ ,$$

which can be used to describe both the left hand side of equation (A.1) and the first term in the mod expression. First consider X(0), the term on the left side of the mod expression. By choosing some q between 0 and L, X(0) can be made an integer. Moreover, if this q were determined when D(0) was the smallest delay, all other X(i) will be positive even with this q subtracted from D(i). In any case, by defining N/L as a positive integer m, the mod expression becomes

$$X(1) = (X(0) + c) \mod m$$

or more generally,

$$X(i+1) = (X(i) + c) \bmod m .$$

This is a special condition of a linear congruential number generator, which, as discussed by Knuth [Ref. 7], is of the form

$$X(n+1) = (aX(n) + c) \bmod m ,$$

where X(0), a, and c are non-negative integers and m is an integer greater than X(0), a, or c. These generators are said to be full cycle, or accept integer values from 0 to m-1, if the following conditions are met:

1. c has no prime factors in common with m.
2. a mod y = 1 for all y which are prime factors of m.
3. a mod 4 = 1 if 4 is a factor of m.

For the component delay case Conditions 2 and 3 are met easily since the parameter a has value 1, and thus has no integral factors other than 1. To check Condition 1 it must be shown that N/L and ((jN/L)-(Y/L)) have no common prime factors. First assume such a factor exists (call it z). For z to be a factor of N/L, N/zL must be an integer. Since N/zL and j are integers, jN/zL must be an integer. If z is a factor of ((jN/L)-(Y/L)), then ((jN/L)-(Y/L))/z must be an integer or (jn/zL)-(Y/zL) must be integer. But it has already been shown that jN/zL is integer so Y/zL must be integer. But if Y/zL and N/zL are both integer for a z

greater than 1, then L was not chosen properly (it was not the largest real number such that N/L and Y/L are both integers). Thus ((jN/L)-(Y/L)) can have no common factors with N/L and Condition 1 must hold. The variables X(i) must assume values 0,1,...,m-1 or have an average value of (m-1)/2. Converting the X variables back to D and using the fact that m is N/L,

$$X = \frac{D - q}{L} = \frac{m - 1}{2} = \frac{(N/L) - 1}{2} = \frac{N - L}{2L}$$

$$D - q = \frac{(N - L)}{2}$$

$$D = \frac{N - L}{2} + q$$

which is the average delay cost per component over the cycle. Since delay is being minimized in this thesis, it is assumed initial conditions will be established such that q is equal to zero.

APPENDIX B

SIMSCRIPT COMPUTER PROGRAM

```
    PREAMBLE
''USES ISSDELY FUNCTION
EVENT NOTICES INCLUDE DELIVERY, START.STAT, END.SIM
  EVERY RECEIPT HAS A GDOC
  EVERY INDUCTION HAS A GNCUST
  EVERY ISSUE HAS AN GITEM
  PRIORITY ORDER IS START.STAT, INDUCTION, ISSUE, DELIVERY,
     RECEIPT, AND END.SIM
DEFINE GDOC, GNCUST AND GITEM AS INTEGER VARIABLES
'' G PREFIX ON VARIABLES TO DENOTE GLOBAL VARIABLE
TEMPORARY ENTITIES
  EVERY COMPONENT HAS A NPARTS, A SDELAY AND A LINE, AND
    BELONGS TO THE REPAIR
'' REPAIR IS THE SET OF ALL ITEMS UNDER REPAIR.
  DEFINE NPARTS AS AN INTEGER VARIABLE
  EVERY REQN HAS A NHA AND MAY BELONG TO THE QUEUE
'' QUEUE IS SET OF ALL REQUISITIONS AWAITING DELIVERY
  DEFINE LINE AND NHA AS INTEGER VARIABLES
PERMANENT ENTITIES
  EVERY CUSTOMER HAS AN INDMIN, AN INDMAX, A PMEAN,
    A NBRRFI, A COSTRATE, A DLY1COST, AND A DLY2COST
'' DLY1COST FOR LOCALLY DELIVERED ITEMS, A DLY2COST FOR OTHERS
THE SYSTEM OWNS THE QUEUE AND THE REPAIR, AND HAS AN ISSDELAY
    RANDOM LINEAR VARIABLE
DEFINE ISSDELAY AS A REAL, STREAM 3 VARIABLE
DEFINE I, PMEAN, K, NBRRFI, TLOAD, FLAG AND KK AS INTEGER
    VARIABLES
DEFINE TRANSCOST, SHIPCOST, STARTSTAT, ENDSIM, AND N AS
    VARIABLES
TALLY SDLY1COST AS THE SUM OF DLY1COST
TALLY SDLY2COST AS THE SUM OF DLY2COST
TALLY MN.N.QU AS THE MEAN AND VAR.N.QU AS THE VARIANCE
    OF TLOAD
END

MAIN
DEFINE SS AS A 1-DIMENSIONAL ARRAY
RESERVE SS() AS 4
DEFINE J, NIT AS INTEGER VARIABLES
READ NIT
READ N.CUSTOMER, SHIPCOST
PRINT 2 LINE WITH N.CUSTOMER, SHIPCOST AS FOLLOWS
N.CUSTOMERS       SHIPPING COSTS
  ***               ****.**
READ ISSDELAY
CREATE EVERY CUSTOMER
PRINT 1 LINE AS FOLLOWS
CUSTOMER        INDMIN    INDMAX    PMEAN     COSTRATE
FOR EACH CUSTOMER, DO
  READ INDMIN(CUSTOMER), INDMAX(CUSTOMER), PMEAN(CUSTOMER),
     COSTRATE(CUSTOMER)
  PRINT 1 LINE WITH CUSTOMER,INDMIN(CUSTOMER),INDMAX(CUSTOMER),
       PMEAN(CUSTOMER),COSTRATE(CUSTOMER) AS FOLLOWS
  ****        ****.**   ****.**   ****.**    ****.**
  LOOP
FOR I=1 TO 4, DO
```

```
'' SAVE ALL RANDOM NUMBER SEEDS
  LET SS(I)=SEED.V(I)
  LOOP
FOR J=1 TO NIT, DO
'' RUN AS MANY ITERATIONS AS DESIRED
READ N, K, STARTSTAT, ENDSIM
FOR I=1 TO 4, DO
'' RESET SEEDS EACH ITERATION
  LET SEED.V(I)=SS(I)
  LOOP
SKIP 2 LINES
FOR EACH CUSTOMER, DO
  CREATE A COMPONENT
  FILE THE COMPONENT IN REPAIR
  LET LINE(COMPONENT)=CUSTOMER
  LET Q=.1
'' USING BINOMIAL WITH P=.1.  PMEAN IS NUMBER OF TRIALS.
  LET NPARTS(COMPONENT)=BINOMIAL.F(PMEAN(CUSTOMER),Q,4)
  LET INDUCT=UNIFORM.F(INDMIN(CUSTOMER),INDMAX(CUSTOMER),2)
''   THIS ALLOWS VARIABILITY FOR TIME BETWEEN INDUCTIONS.
''   IN THESIS MAX VALUE AND MIN VALUE WAS ALWAYS THE SAME.
  LET SDELAY(COMPONENT)=0
  FOR I=1 TO NPARTS(COMPONENT), DO
    CREATE A REQN
    LET NHA(REQN)=COMPONENT
   LET IWAIT=ISSDELAY
   LET WAIT =IWAIT
       IF IWAIT LE 7 AND IWAIT GE 2.5
'' IF IWAIT IS GT 7, THE ISSUE IS NON-LOCAL. IF IT IS
'' LT 2.5, ISSUE IS FROM LOCAL WAREHOUSE.  IN EITHER CASE
'' LOCAL DELIVERY SYSTEM IS NOT USED.
       SCHEDULE AN ISSUE GIVEN REQN IN WAIT DAYS
       ELSE SCHEDULE A RECEIPT GIVEN REQN IN WAIT DAYS
    REGARDLESS
    LOOP
  SCHEDULE AN INDUCTION GIVEN CUSTOMER IN INDUCT/2 DAYS
  LOOP
SCHEDULE AN END.SIM AT ENDSIM
SCHEDULE A START.STAT AT STARTSTAT
LET KK=K
LET K=9999
'' THIS K VALUE SERVES AS FLAG THAT DELIVERY EVERY N DAY
'' OPTION IS IN USE.  KK SAVES K VALUE.
SCHEDULE A DELIVERY IN N DAYS
START SIMULATION
''   REPEAT FOR DELIVER EVERY K REQN OPTION.
LET TIME.V=0
IF KK NE 0
    FOR I=1 TO 4, DO
      LET SEED.V(I)=SS(I)
      LOOP
FOR EACH CUSTOMER, DO
  CREATE A COMPONENT
  FILE THE COMPONENT IN REPAIR
  LET LINE(COMPONENT)=CUSTOMER
  LET Q=.1
  LET NPARTS(COMPONENT)=BINOMIAL.F(PMEAN(CUSTOMER),Q,4)
  LET INDUCT=UNIFORM.F(INDMIN(CUSTOMER),INDMAX(CUSTOMER),2)
  LET SDELAY(COMPONENT)=0
  FOR I=1 TO NPARTS(COMPONENT), DO
    CREATE A REQN
    LET NHA(REQN)=COMPONENT
   LET IWAIT=ISSDELAY
   LET WAIT =IWAIT
       IF IWAIT LE 7 AND IWAIT GE 2.5
       SCHEDULE AN ISSUE GIVEN REQN IN WAIT DAYS
       ELSE SCHEDULE A RECEIPT GIVEN REQN IN WAIT DAYS
    REGARDLESS
```

```
    LOOP
  SCHEDULE AN INDUCTION GIVEN CUSTOMER IN INDUCT/2 DAYS
  LOOP
    SCHEDULE AN END.SIM IN ENDSIM DAYS
    SCHEDULE A START.STAT IN STARTSTAT DAYS
    LET K=KK
    START SIMULATION
    LET TIME.V=0
  REGARDLESS
  LOOP
STOP
END

EVENT DELIVERY
LET FLAG=0
LET TLOAD=N.QUEUE
IF K EQ 9999
  SCHEDULE A DELIVERY IN N DAYS
  REGARDLESS
LET TRANSCOST=TRANSCOST+SHIPCOST
FOR EACH REQN IN THE QUEUE, DO
  REMOVE THE REQN FROM THE QUEUE
  LET COMPONENT=NHA(REQN)
  LET NPARTS(COMPONENT)=NPARTS(COMPONENT)-1
  IF NPARTS(COMPONENT) LE   0
    LET DLY1COST(LINE(COMPONENT))=(TIME.V-SDELAY(COMPONENT))*
          COSTRATE(LINE(COMPONENT))
    LET NBRRFI(LINE(COMPONENT))=NBRRFI(LINE(COMPONENT))+1
    REMOVE THE COMPONENT FROM REPAIR
    DESTROY THE COMPONENT
  REGARDLESS
  DESTROY THE REQN
  LOOP
RETURN
END

EVENT START.STAT
FOR EACH CUSTOMER, RESET THE TOTALS OF DLY1COST
FOR EACH CUSTOMER, RESET THE TOTALS OF DLY2COST
RESET THE TOTALS OF TLOAD
LET TRANSCOST=0
FOR EACH CUSTOMER, DO
  LET NBRRFI(CUSTOMER)=0
  LET DLY1COST(CUSTOMER)=0
  LET DLY2COST(CUSTOMER)=0
  LOOP
RETURN
END

EVENT END.SIM
FOR EACH REQN IN THE QUEUE, DO
  REMOVE THE REQN FROM THE QUEUE
  DESTROY THE REQN
  LOOP
FOR EACH COMPONENT IN REPAIR, DO
  REMOVE THE COMPONENT FROM REPAIR
  DESTROY THE COMPONENT
  LOOP
LET TC=0
FOR EACH RECEIPT IN EV.S(I.RECEIPT), DO
  CANCEL THE RECEIPT
  LOOP
FOR EACH DELIVERY IN EV.S(I.DELIVERY), DO
  CANCEL THE DELIVERY
  LOOP
FOR EACH ISSUE IN EV.S(I.ISSUE), DO
  CANCEL THE ISSUE
  LOOP
```

```
FOR EACH INDUCTION IN EV.S(I.INDUCTION), DO
  CANCEL THE INDUCTION
  LOOP
SKIP 2 LINES
PRINT 1 LINE WITH STARTSTAT, ENDSIM THUS
   STARTSTAT=***.**   ENDSIM=****.**
IF K EQ 9999
  PRINT 3 LINES WITH N, TRANSCOST, MN.N.QU, AND VAR.N.QU THUS
  RESULTS FOR N=***.** CASE.  TOTAL TRANS COST IS ****.**
               LOAD MEAN=****.**   LOAD VARIANCE=***.**
  CUSTOMER  NBRRFI   AVE LOC D COST    AVE NON-LOC D COST
ELSE
  PRINT 3 LINES WITH K, TRANSCOST, MN.N.QU, AND VAR.N.QU THUS
  RESULTS FOR K=*** CASE.  TOTAL TRANSPORTATION COST= ****.**
               LOAD MEAN=****.**    LOAD VARIANCE=***.**
  CUSTOMER  NBRRFI   AVE LOC D COSTS   AVE NON-L D COSTS
REGARDLESS
FOR EVERY CUSTOMER, DO
  PRINT 1 LINE WITH CUSTOMER, NBRRFI(CUSTOMER),
    SDLY1COST(CUSTOMER)/(ENDSIM-STARTSTAT),
    SDLY2COST(CUSTOMER)/(ENDSIM-STARTSTAT) THUS
    ***      ****          ****.**          ****.**
  LET D1=SDLY1COST(CUSTOMER)+D1
  LET D2=SDLY2COST(CUSTOMER)+D2
  LOOP
LET TC=(TRANSCOST+D1+D2)/(ENDSIM-STARTSTAT)
SKIP 1 LINE
LET PER=ENDSIM-STARTSTAT
PRINT 2 LINES WITH D1/PER, D2/PER, TC THUS
  AVE LOCAL DEL DELAY= ****.** AVE OTHER DELAY= ****.**
  AVE TOTAL=****.**
LET TRANSCOST=0
LET D1=0
LET D2=0
RETURN
END

EVENT INDUCTION GIVEN NCUST
  DEFINE NCUST AS INTEGER VARIABLE
  LET CUSTOMER=NCUST
  CREATE A COMPONENT
  FILE THE COMPONENT IN REPAIR
  LET LINE(COMPONENT)=CUSTOMER
  LET Q=.1
  LET NPARTS(COMPONENT)=BINOMIAL.F(PMEAN(CUSTOMER), Q,4)
  LET INDUCT=UNIFORM.F(INDMIN(CUSTOMER),INDMAX(CUSTOMER),2)
  LET SDELAY(COMPONENT)=TIME.V + INDUCT/2
  FOR I=1 TO NPARTS(COMPONENT), DO
    CREATE A REQN
    LET NHA(REQN)=COMPONENT
    LET IWAIT=ISSDELAY
    LET WAIT =INDUCT/2+IWAIT
    IF IWAIT LE 7 AND IWAIT GE 2.5
       SCHEDULE AN ISSUE GIVEN REQN IN WAIT DAYS
       ELSE SCHEDULE A RECEIPT GIVEN REQN IN WAIT DAYS
    REGARDLESS
    LOOP
SCHEDULE AN INDUCTION GIVEN CUSTOMER IN INDUCT DAYS
RETURN
END

EVENT ISSUE GIVEN ITEM
DEFINE ITEM AS INTEGER VARIABLE
LET REQN=ITEM
FILE THE REQN IN THE QUEUE
IF FLAG EQ 0 AND N.QUEUE GE K
  LET FLAG=1
  SCHEDULE A DELIVERY NEXT
```

```
REGARDLESS
LET COMPONENT=NHA(REQN)
RETURN
END

EVENT RECEIPT GIVEN DOC
DEFINE DOC AS INTEGER VARIABLE
LET REQN=DOC
LET COMPONENT=NHA(REQN)
LET NPARTS(COMPONENT)=NPARTS(COMPONENT)-1
IF NPARTS(COMPONENT) LE 0
  LET DLY2COST(LINE(COMPONENT))=(TIME.V-SDELAY(COMPONENT))*COSTRATE
    (LINE(COMPONENT))
  LET NBRRFI(LINE(COMPONENT))=NBRRFI(LINE(COMPONENT))+1
  REMOVE THE COMPONENT FROM REPAIR
  DESTROY THE COMPONENT
  REGARDLESS
DESTROY THE REQN
RETURN
END
```

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